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# Bharati Vidyapeeth's <br> Institute of Computer Applications and Management (BVICAM), A-4, Paschim Vihar, New Delhi-63 <br> FIRST SEMESTER [MCA] Internal Examination, December 2023 

Paper Code: MCA-101
Subject: Discrete Structures
Time: 2 Hours
Maximum Marks: 45
Note: Attempt THREE questions in all. Question No. 1 is compulsory, and attempt one question from each unit.

| 1. | Answer all the following questions briefly: - |  | $1.5 \times 10=15$ |  |
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|  | (a) | Use the properties of sets to prove that for all the sets $A$ and $B, A-(A \cap B)=A-B$ |  | CO1 |
|  | (b) | Let $S$ be the set of all points in a plane. Let R be a relation such that for any two points $a$ and $b$ : $(a, b$ belongs to $R)$ if $b$ is within 2 centimeters from $a$. Show that R is an equivalence relation. |  | CO1 |
|  | (c) | In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme, 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made? |  | CO 2 |
|  | (d) | Develop DNF of the $\sim(p \mathrm{~V} q)<->(\mathrm{p} \wedge \mathrm{q})$ |  | CO 2 |
|  | (e) | Show by induction that the sum of the cubes of three consecutive integers is divisible by 9 . |  | CO1 |
|  | (f) | Develop the existential formula for the sentence" Not all rainy days are cold" considering $\mathrm{R}(\mathrm{d})$ : Rainy days and $\mathrm{C}(\mathrm{x})$ : Cold days |  | CO 2 |
|  | (g) | In a group of students, there are 6 boys and 4 girls. Out of 10 students, 4 students have to be selected. Find out how many different ways the students can be selected such that at least one boy should be selected? |  | CO 2 |
|  | (h) | Write fog, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x)=8 x^{3}$ and $g(x)=x^{1 / 3}$ |  | CO 2 |
|  | (i) | Find out the number of ways that the letters of the word "DETAIL" can be arranged such that the vowels must occupy odd positions. |  | CO 2 |
|  | (j) | If f is an invertible function, defined as $\mathrm{f}(\mathrm{x})=(3 x-4) / 5$, then write $\mathrm{f}^{-1}(\mathrm{x})$. |  | CO1 |
| UNIT - I |  |  |  |  |
| 2. | (a) | i) Assuming repetitions are not allowed, how many 4 digit numbers can be formed from 6 digits $1,2,3,5,7,8$ ? <br> ii) How many of these are less than 4000 ? <br> iii)How many in part i) are even? <br> iv) How many in part i) contain both 3 and 5? | 5 | CO1 |
|  | (b) | Consider $\mathrm{A}=\{4,5,6,7\}$ and $\mathrm{R}=\{(4,4),(5,5),(6,6),(7,7),(4,6),(6,4)\}$ | 5 | CO 2 |


|  |  | Evaluate <br> i) Reflexive closure <br> ii) Symmetric closure <br> iii) Transitive closure |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | (c) | Find the solution of recurrence relation $\mathrm{a}_{\mathrm{r}}=\mathrm{a}_{\mathrm{r}-1}+2 \mathrm{a}_{\mathrm{r}-2}$ with $\mathrm{a}_{0}=2$ and $\mathrm{a}_{1}=7$ | 5 | CO3 |
| 3. | (a) | A survey on of 1000 people, 595 like metro channel, 595 like Star movies, 550 like Zee TV, 395 like both metro and star, 350 like metro and Zee, 400 like Star and Zee, 250 like all three. How many <br> i) Do not like metro, star and Zee <br> ii) Like Metro and do not like Star and Zee <br> iii) Like Zee and do not like Metro and Star <br> iv) Like only Zee <br> v) Like atleast one channel | 5 | CO1 |
|  | (b) | Justify by giving example of relation R1,R2,R3 and R4 on $A=\{4,5,6,7,8\}$ having property <br> i) R1 is reflexive and symmetric but not transitive <br> ii) R2 is symmetric and antisymmetric <br> iii) R3 is antisymmetric but not reflexive <br> iv) R4 is transitive but not reflexive | 5 | CO 2 |
|  | (c) | $\begin{aligned} & \text { Let } \mathrm{A}=\{\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{~d}, \mathrm{e}\} \\ & \text { and } \mathrm{R}=\{(\mathrm{a}, \mathrm{~b}),(\mathrm{a}, \mathrm{a}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{~b}),(\mathrm{c}, \mathrm{c}),(\mathrm{d}, \mathrm{~d}),(\mathrm{d}, \mathrm{e}),(\mathrm{e}, \mathrm{~d}),(\mathrm{e}, \mathrm{c})\} \\ & \mathrm{S}=\{(\mathrm{a}, \mathrm{a}),(\mathrm{b}, \mathrm{~b}),(\mathrm{c}, \mathrm{c}),(\mathrm{d}, \mathrm{~d}),(\mathrm{e}, \mathrm{e}),(\mathrm{a}, \mathrm{c}),(\mathrm{c}, \mathrm{a}),(\mathrm{d}, \mathrm{e}),(\mathrm{e}, \mathrm{~d})\} \text { be equilence relations } \\ & \text { on A. Determine the partitions corresponding to i) } \mathrm{R}^{-1} \text { ii) R U S iii) } \mathrm{R} \cap \mathrm{~S} \end{aligned}$ | 5 | CO1 |
| UNIT - II |  |  |  |  |
| 4. | (a) | Prove the following without truth table <br> i) $(\mathrm{p} V \mathrm{q})->\sim \mathrm{r}, \mathrm{r} V \mathrm{t}, \mathrm{p} \mid-\mathrm{t}$ <br> ii) $\left(\mathrm{p}^{\wedge} \mathrm{q}\right)->\mathrm{r},(\mathrm{r}->\mathrm{q}),(\mathrm{r}->\mathrm{q})->\left(\mathrm{q}^{\wedge} \mathrm{r}\right) \mid-\left(\mathrm{p}^{\wedge} \mathrm{q}\right)->\left(\mathrm{q}^{\wedge} \mathrm{r}\right)$ <br> iii) $p \vee q, q->r, r^{\wedge} s, p->s, p \mid--s$ | 5 | CO 2 |
|  | (b) | Solve the recurrence relation $a_{r+2}-2 a_{r+1}+a_{r}=2^{r}$ and find the particular solution if $\mathrm{a}_{0}=2$ and $\mathrm{a}_{1}=1$ | 5 | CO 2 |
|  | (c) | Solve by induction $1^{3}+2^{3}+3^{3+} \ldots . . n^{3}=[n(n+1) / 2]^{2}$ | 5 | CO3 |
| 5. | (a) | Translate the following into symbolic form and test the validity <br> i) If 6 is even then 2 does not divide 7 . Either 5 is not prime or 2 divides 7. But 5 is prime, therefore 6 is odd <br> ii) If it rains then it will be cold. If it is cold then I shall stay at home. Since it rains, therefore I shall stay at home | 5 | CO3 |


|  | (b) | Solve the recurrence relation $\mathrm{a}_{\mathrm{r}+2}-5 \mathrm{a}_{\mathrm{r}+1}+6 \mathrm{a}_{\mathrm{r}}=2$ and find the particular <br> solution if $\mathrm{a}_{0}=1$ and $\mathrm{a}_{1}=2$ | 5 | CO 2 |
| :--- | :--- | :--- | :--- | :--- |
|  | (c)Solve by Induction $12 / 1.3 \quad+22^{2} / 3.5 \quad+\ldots \ldots \ldots+n^{2} /(2 n-1) \cdot(2 n$ <br> $+1)=n(n+1) / 2(2 n+1)$ | 5 | CO 3 |  |

