Bharati Vidyapeeth's Institute of Computer Applications and Management (BVICAM),

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FIRST SEMESTER [MCA] Internal Examination, December 2023

Paper Code: MCA-101

Subject: Discrete Structures

Time: 2 Hours

Maximum Marks: 45

Note: Attempt THREE questions in all. Question No. 1 is compulsory, and attempt one question from each unit.

| 1. | Answer all the following questions briefly: - $1.5 \times 10 = 1$ | | | | | | | |
|----|---|---|---|-----|--|--|--|--|
| | (a) Use the properties of sets to prove that for all the sets A and B, $A - (A \cap B) = A - B$ | | | | | | | |
| | (b) Let S be the set of all points in a plane. Let R be a relation such that for | | | | | | | |
| | | two points a and b: (a, b belongs to R) if b is within 2 centimeters from a | | | | | | |
| | | Show that R is an equivalence relation. | | | | | | |
| | (c) In a small village, there are 87 families, of which 52 families have at most 2 cl | | | | | | | |
| | In a rural development programme, 20 families are to be chosen for assistance, | | | | | | | |
| | | which at least 18 families must have at most 2 children. In how many ways can the choice be made? | | | | | | |
| | (d) Develop DNF of the \sim (p V q) <-> (p ^ q) | | | | | | | |
| | (e) Show by induction that the sum of the cubes of three consecutive integers is | | | | | | | |
| | | divisible by 9. | | | | | | |
| - | (f) Develop the existential formula for the sentence" Not all rainy days are cold | | | | | | | |
| | considering R(d) : Rainy days and C(x): Cold days | | | | | | | |
| | (g) In a group of students, there are 6 boys and 4 girls. Out of 10 students, 4 | | | | | | | |
| | | students have to be selected. Find out how many different ways the students | | | | | | |
| | | can be selected such that at least one boy should be selected? | | | | | | |
| | (h) | Write fog, if f: R \rightarrow R and g: R \rightarrow R are given by f(x) = 8x ³ and g(x) = x ^{1/3} | | | | | | |
| | (i) | Find out the number of ways that the letters of the word "DETAIL" can be arranged | | | | | | |
| | | such that the vowels must occupy odd positions. | | | | | | |
| | (j) | If f is an invertible function, defined as $f(x) = (3x-4)/5$, then write $\overline{f^{-1}(x)}$. | | | | | | |
| | | UNIT – I | | | | | | |
| 2. | (a) | i) Assuming repetitions are not allowed, how many 4 digit numbers | 5 | CO1 | | | | |
| | | can be formed from 6 digits 1, 2, 3, 5 ,7 ,8? | | | | | | |
| | | ii) How many of these are less than 4000? | | | | | | |
| | | 111)How many in part i) are even? | | | | | | |
| | | iv) How many in part i) contain both 3 and 5? | | | | | | |
| | (b) | Consider A= $\{4,5,6,7\}$ and R= $\{(4,4),(5,5),(6,6),(7,7),(4,6),(6,4)\}$ | 5 | CO2 | | | | |

| | | Evaluate | | | | | | |
|----|--|--|---|-----|--|--|--|--|
| | | i) Reflexive closure | | | | | | |
| | | ii) Symmetric closure | | | | | | |
| | | iii) Transitive closure | | | | | | |
| | (c) | Find the solution of recurrence relation $a_r = a_{r-1} + 2a_{r-2}$ with $a_0 = 2$ and | | | | | | |
| | | a ₁ =7 | | | | | | |
| 3. | (a) | A survey on of 1000 people, 595 like metro channel, 595 like Star | 5 | CO1 | | | | |
| | movies, 550 like Zee TV, 395 like both metro and star,350 like met | | | | | | | |
| | | and Zee, 400 like Star and Zee, 250 like all three. How many | | | | | | |
| | | i) Do not like metro, star and Zee | | | | | | |
| | | ii) Like Metro and do not like Star and Zee | | | | | | |
| | iii) Like Zee and do not like Metro and Star | | | | | | | |
| | iv) Like only Zee | | | | | | | |
| | | v) Like atleast one channel | | | | | | |
| | (b) | Justify by giving example of relation R1,R2,R3 and R4 on A={4,5,6,7,8 } | 5 | CO2 | | | | |
| | | having property | | | | | | |
| | | i) R1 is reflexive and symmetric but not transitive | | | | | | |
| | | ii) R2 is symmetric and antisymmetric | | | | | | |
| | | iii) R3 is antisymmetric but not reflexive | | | | | | |
| | | iv) R4 is transitive but not reflexive | | | | | | |
| | (c) | Let $A = \{a,b,c,d,e\}$ | 5 | CO1 | | | | |
| | | and R= $\{(a,b),(a,a),(b,a),(b,b),(c,c),(d,d),(d,e),(e,d),(e,c)\}$ | | | | | | |
| | | $S=\{(a,a),(b,b),(c,c),(d,d),(e,e),(a,c),(c,a),(d,e),(e,d)\}$ be equilence relations | | | | | | |
| | | on A. Determine the partitions corresponding to i) R ⁻¹ ii) R U S iii) R \cap S | | | | | | |
| | | UNIT – II | | | | | | |
| 4. | (a) | Prove the following without truth table | 5 | CO2 | | | | |
| | | i) (p V q) ->~r, r V t, p -t | | | | | | |
| | | ii) (p^q)->r,(r->q),(r->q)->(q^r) (p^q)->(q^r) | | | | | | |
| | | iii) pVq, q->r ,r^s, p->s, p s | | | | | | |
| | (b) | Solve the recurrence relation a_{r+2} . $2a_{r+1} + a_r = 2^r$ and find the particular | 5 | CO2 | | | | |
| | | solution if $a_0 = 2$ and $a_1 = 1$ | | | | | | |
| | (c) | Solve by induction $1^3 + 2^3 + 3^3 + \dots + n^3 = [n(n+1)/2]^2$ | 5 | CO3 | | | | |
| 5. | (a) | Translate the following into symbolic form and test the validity | 5 | CO3 | | | | |
| | | i) If 6 is even then 2 does not divide 7. Either 5 is not prime or 2 divides | | | | | | |
| | | 7. But 5 is prime, therefore 6 is odd | | | | | | |
| | | ii) It it rains then it will be cold. If it is cold then I shall stay at home. | | | | | | |
| | | Since it rains, therefore I shall stay at home | | | | | | |

| (b) | Solve the recurrence relation a_{r+2} . $5a_{r+1} + 6a_r = 2$ and find the particular solution if $a_0 = 1$ and $a_1 = 2$ | | | | | | 5 | CO2 |
|-----|---|-----------------------|---------------------|---|---------------------|------------------------------|---|-----|
| (c) | Solve by +1) = n(n+1)/ | Induction (2(2n+1) | 1 ² /1.3 | + | 2 ² /3.5 | ++n ² /(2n-1).(2n | 5 | CO3 |