

2023 – 2025 Batch Question Paper (External)

MCA-101 EXAMINATION

FIRST SEMESTER (MCA) JANUARY-2024

Paper Code: MCA-101

Subject: Discrete Structures

Time: 3 Hours

Maximum Marks: 60

Note: Attempt five questions in all including question no. 1 which is compulsory. Select one question from each unit.

Q1 Answer all the following questions briefly. (2x10=20)

- (a) Show that $(A-B) - C = A - (B \cup C)$
- (b) Show that if any five integers from 1 to 8 are chosen, then atleast two of them will have a sum 9.
- (c) Solve by mathematical induction $1(1) + 2(2) + 3(3) + \dots + n(n) = (n+1) \cdot n / 2$. In a group of students, there are 7 boys and 8 girls. Out of 15 students, 6 students have to be selected. Find out how many different ways the students can be selected such that at least one boy should be selected?
- (d) Show that the 4 fourth roots of unity form a group with respect to multiplication.
- (e) Show that $2^{30} - 1$ (mod 11) by Fermat little theorem.
- (f) Justify by giving example of relation R1, R2, R3 and R4 on $A = \{4, 5, 6, 7, 8\}$ having property
 - (h) R1 is reflexive and symmetric but not transitive
 - (i) R2 is symmetric and antisymmetric
- (g) Consider the Boolean algebra D30. Determine all the Boolean sub-algebra of D30 having atleast four element.
- (h) Show that inverse of an element a in the group is unique.
- (i) Prove that complete planar graph K4 is planar.

UNIT - I

- Q2(a) Among the first 500 positive integers:
- i. Determine the integers which are not divisible by 2, nor by 3, nor by 5.
 - ii. Determine the integers which are exactly divisible by one of them.
- (b) i. Assuming repetitions are not allowed, how many 4 digit numbers can be formed from digits 1, 2, 3, 5, 7, 8
- ii. How many of these are less than 4000?
 - iii. How many in part i) are odd?
 - iv. How many in part i) contain both 2 and 5?
- Q3(a) Let $A = \{1, 2, 3, 4\}$ and $R = \{(2, 1), (2, 3), (3, 2), (4, 3)\}$. Find the
 - i. Reflexive closure of R
 - ii. Symmetric closure of R
 - iii. Transitive closure of R using warshall's algo.
 Find the particular solution of the difference equation $a_{n+2} + a_{n+1} + a_n = r^2$
- Q4(a) Prove the following without truth table
- i. $(p \vee q) \rightarrow \neg r \vee p \vee \neg r$
 - ii. $p \rightarrow \neg q \vee r, \neg(r \vee \neg s) \rightarrow p \rightarrow s$
 - iii. $(q \rightarrow r) \wedge (s \rightarrow r) \wedge (q \rightarrow s) \wedge (p \vee \neg r) \vee (p \vee \neg s) \rightarrow p$
- Draw Hasse diagram for D100. Find GUB and LUB for $B = \{10, 20\}$ and $C = \{5, 10, 20, 25\}$. Is D100 is Finite Boolean algebra. Justify.

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- Q5(a) If L is a distributive lattice, for any $a, b, c \in L$, then show that
 - i. $a \vee b = a \vee c$ and $a \wedge b = a \wedge c \Rightarrow b = c$
 - ii. $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$
 Draw K map and simplify the following Boolean expression

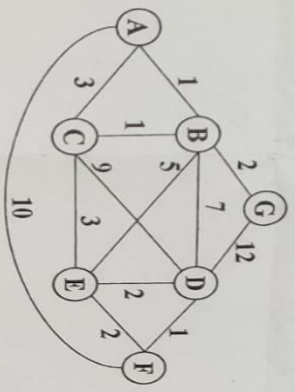
$$ABC'D + AB'CD + ABC'D + AB'CD$$

UNIT - III

- Q6(a) Prove that the order of each subgroup of a finite group G is a divisor of the order of the group G.
- (b) Determine $\gcd(a, b)$. Find s and t such that $\gcd(a, b) = sa + tb$, where $a=45$ and $b=126$

UNIT - IV

- Q7(a) Prove that the necessary and sufficient condition that a non-empty subset H of a Group G be a subgroup is a $e \in H, b \in H \Rightarrow ab^{-1} \in H$
- (b) Consider two distinct prime numbers $p=17$ and $q=19$. Apply the RSA algorithm to find the public key and private key.
- Q8 (a) An undirected tree has a 1 vertex with degree 1, 2 vertices with degree 2 each, 3 vertices with degree 3 each, and n vertices with degree n each. Prove that no such tree exists.
- (b) Use Dijkstra's algorithm to find the shortest path between a and f in the given graph.



- Q9 (a) State and Prove 5 color theorem.
- (b) Give an example of a graph which contains
 - i. an Eulerian circuit and a Hamiltonian circuit.
 - ii. an Eulerian circuit and a Hamiltonian circuit that are distinct.
 - iii. a Hamiltonian circuit, but not an Eulerian circuit.
 - iv. a Hamiltonian circuit, but not an Eulerian circuit.
 - v. Neither an Eulerian circuit, nor a Hamiltonian circuit.

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