

END TERM EXAMINATION

FIRST SEMESTER [MCA] DECEMBER 2015- JANUARY 2016

Paper Code: MCA 105

Subject: Discrete Mathematics
(Batch: 2015)

Time : 3 Hours

Maximum Marks : 75

Note: Attempt any five questions including Q. No. 1 which is compulsory.
Select one question from each unit.

- Q1. a) Show that a set of n elements can have 2^n subsets. (2.5x10=25)
b) Define binary relation. How many binary relations are there on a set A with n elements?
c) How many ways are there to arrange 7- sign and 5+ sign, such that no two +sign are together?
d) The set $P(\{a,b,c\})$ is partially ordered with respect to the subset relation. Find a chain of length 3 in P .
e) Show that D_{20} is not a finite Boolean algebra with the partial order of divisibility.
f) Find the solution of recurrent relation $a_n = 3a_{n+1}$ where $a_0 = 1$.
g) What are applications of number theory in computer science?
h) Show that any subgroup of a cyclic group is cyclic.
i) Giving graphical representation discuss seven bridge problem. Was it possible for a citizen to make a tour of the city and across each bridge exactly twice? Give reasons.
j) Define Hamiltonian graph with example.

Unit-I

- Q2. a) Let Z be the set of all integers and R be a relation defined on Z such that for any $a, b \in Z$, aRb if and only if $ab \geq 0$. Is R an equivalence relation? (3.5)
b) Find the number of integers between 1 and 100 that are divisible by any of the integer 2,3,5,7. (4)
c) Show by mathematical induction
$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \forall n \in N. \quad (5)$$

- Q3. a) Without using truth table, prove the following: (6.25)
 $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv (p \wedge q)$
b) Define function. Find the inverse of function $f(x) = \frac{1}{x-1}, x \neq 1$. (6.25)

Unit-II

- Q4. a) Let (L, \leq) be a bounded distributive lattice with 1 and 0 as unit and zero elements of L respectively. (6.25)
i) Prove the DeMorgan's Law.
ii) Show that if the complement of an element in L exists then it is unique.
b) Simplify $y = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$ using K-map. (6.25)

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- Q5. a) Let L_1 be the lattice D_6 (divisor of 6) = {1,2,3,6} and let L_2 be the lattice $(P(S), \subseteq)$ where $S = \{a, b\}$. Show that two lattices are isomorphic. (6.25)
- b) Minimize the Boolean expression $f(x,y,z, w) = \sum (0, 3,4,5, 7)$ and $d(x,y,z, w) = \sum (8,9,10,11,12,13,14,15)$. (6.25)

Unit-III

- Q6. a) Explain Euclidean algorithm to find the gcd of two numbers by taking example. (6.25)
- b) Find the code words generated by the parity-check matrix given below. (6.25)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \text{ when the encoding function is } E: B^3 \rightarrow B^6.$$

- Q7. a) Let $(G, *)$ be a group. Let $H = \{ a : a \in G \text{ and } a*b = b*a \text{ for all } b \in G \}$. Show that H is a normal subgroup. (6.25)
- b) Let $(G, .)$ be a group. Let $(H, .)$ be a subgroup of $(G, .)$. Show that $G = H \cup Ha \cup Hb$, where $a, b \in G$. (6.25)

Unit-IV

- Q8. a) Differentiate between
i) Graph and Tree (7.5)
ii) Sub graphs and isomorphic graph
iii) Connected and complete graph

b) Let $G=(V,E)$ be an undirected graph with edges then show that $2e = \sum \text{deg}(v), v \in V$. (5)

Q9. a) Explain inorder, preorder and postorder tree traversals with the help of an example. (7.5)

b) Show that a graph is two colorable if and only if it is a bipartite graph. (5)
