

END TERM EXAMINATION

FIRST SEMESTER [MCA] DECEMBER 2013

Paper Code: MCA105

Subject: Discrete Mathematics

Time: 3 Hours

Maximum Marks: 60

Note: Attempt any five questions including Q.no.1 which is compulsory.

Select one question from each unit.

Q1. Attempt **any ten** of the following:

[10 x 2=20]

- How many diagonals are there in a regular decagon.
- Prove that $p \rightarrow (p \vee q)$ is a tautology.
- The set $P(\{a,b,c\})$ is partially ordered with respect to the subset relation. Find a chain of length 3 in P .
- Find the solution of recurrence relation $a_n = 3a_{n-1} + 1$ where $a_0 = 1$.
- Prove that if $\gcd(a,b) = 1$ then $\gcd(a^2, b^2) = 1$.
- Consider $(m, 3m)$ encoding function, where $m=4$. For received word 011010011111 an error will occur or not.
- Give 2 ways to represent a graph in computer.
- Define hamiltonian graph with example.
- Show that any subgroup of a cyclic group is cyclic.
- Show that if any 5 numbers from 1 to 8 are chosen, then two of them will add up to 9.
- How many ways are there to arrange 7 $-$ sign and 5 $+$ sign, such that no two $+$ sign are together.

UNIT –I

Q2. a) knight is a person who always tell truth and knave always lie. We have two people A and B such that

A says “B is a Knight”, B says “the two of us are opposite”

What are A and B ? [3]

b) Let Z be the set of all integers and R be a relation defined on Z such that for any $a, b \in Z$, aRb if and only if $ab \geq 0$. Is R an equivalence relation ? [4]

c) Show that a set of n elements can have 2^n subsets. [3]

Q3.a) Prove that $|xy| = |x||y|$ is true for all real numbers x and y . [3]

b) Define function. Find the inverse of $f(x) = 2(x-2)^2 + 3$ for all $x \leq 2$. [4]

c) Find the number of integers between 1 and 100 that are divisible by any of the integer 2, 3, 5, 7. [3]

UNIT -II

Q4.a) solve the difference equation

$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r$ for $r \geq 2$ with the boundary conditions $a_0 = 1$ and $a_1 = 1$. [5]

b) let L_1 be the lattice D_6 (divisor of 6) = {1, 2, 3, 6} and let L_2 be the lattice $(P(S), \subseteq)$ where $S = \{a, b\}$. Show that two lattices are isomorphic. [5]

Q5.a) simplify $y = \sum m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$ using K-map. [5]

b) Compute $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = 8f(n/2) + n^2$ with $f(1) = 1$. [5]

UNIT -III

Q6.a) Let $(G, *)$ be a group. Let $H = \{a : a \in G \text{ and } a*b = b*a \text{ for all } b \in G\}$.

Show that H is a normal subgroup. [5]

b) Is 8792002627912 a valid universal code. Explain. [3]

c) Solve $34x=60(\text{mod}98)$ [2]

Q7.a) A code G contains 16 code words: 0000000, 1111111, 1101000 and all its cyclic shifts, 0010111 and all its cyclic shifts. show that (G,) is a group code. Set up the coset table to show that G can correct all single transmission errors. [5]

b) Encrypt the word 'BOOK' and 'PARK' using ceaser cipher system
 $f(p)=p+3(\text{mod}26)$. [5]

UNIT -IV

Q8.a) Define Eulerian graph. Prove that a non empty connected graph is eulerian if and only if its vertices are all of even degree. [4]

b) Differentiate between [3*2=6]

- i. Graph and Tree
- ii. Sub graphs and isomorphic graph
- iii. Connected and complete graph

Q9.a) Prove that a planar graph G is 5 colorable. [5]

b) Explain inorder, preorder and postorder tree traversals with the help of an example. [5]