SET Theory and RELATIONS

1)Show that the following argument is valid:
$S1$: All my tin objects are saucepans.
$S2$: I find all your presents very useful.
$S3$: None of my saucepans is of the slightest use.
S : Your presents to me are not made of tin.

2) For each of the following conditions what relation must be held between sets?

a) $(A \wedge B)^c = B^c$

b) $(A \wedge B) = (A \wedge C)$ and $(A^c \wedge B) = (A^c \wedge C)$

3) Use strong form of mathematical induction to prove that every natural number n > 2 is either prime or product of primes.

4) If number of students who got Grade A in first examination is equal to that of in second examination. If total number of students who got Grade A in exactly one examination is 40, and 4 students did not get Grade A in either examinations, determine the no. of students who got Grade A in a)first exam only, who got Grade A in b)second exam only and who got Grade A in c)both the exams?

5) The 60,000 fans who attended the homecoming football game bought up all the paraphernalia for their cars. Altogether, 20,000 bumper stickers, 36,000 window decals. and 12,000 key rings were sold. We know that 52,000 fans bought at least one item and no one bought more than one of a given item. Also, 6000 fans bought both decals and key rings, 9000 bought both decals and bumper stickers, and 5000 bought both key rings and bumper stickers. (a) How many fans bought all three items? (b) How many fans bought exactly one item? (c) Someone questioned the accuracy of the total number of purchasers, 52,000 (given that all the other numbers have been confirmed to be correct). This person claimed the total number of purchasers to be either 60,000 or 44,000. How do you dispel the claim?

6) Seventy-five children went to an amusement park where they can ride on the merry-go-round, roller coaster, and ferns wheel. It is known that 20 of them have taken all three rides, and 55 of them have taken at least two of the three rides. Each ride costs \$0.50, and the total receipt of the amusement park was \$70. Determine the number of children who did not try any of the rides.

7)Suppose C is a collection of relations S on a set A and let T be the intersection of the relation S, that is, $T = \bigcap(S: S \in C)$. Prove: (a) If every S is symmetric, then T is symmetric. (b) If every S is transitive, then T is transitive.

8) Let R be a relation on a set A, and let P be a property of relations, such as, symmetry and transitivity. Then P will be called R-closable if P satisfies the following two conditions:

1. There is a P-relation S containing R.

2. The intersection of P-relations is a P-relation.

(a) Show that symmetry and transitivity are R-closable for any relation R. (b) Suppose P is R-closable. Then P(R), the P-closure of R. is the intersection of all P-relations S containing R, that is,

$P(R) = \bigcap(S\text{: S is a P-relation and } R \subseteq S)$

9) Consider the set Z of integers. Define aRb by $b = a^r$ for some positive integer r. Show that R is a partial order on Z, that is, show that R is: (a) reflexive; (b) antisymmetric, (c) transitive.