Nonlinear Circuit Modeling Using Volterra Series

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Abstract - In this paper Volterra series has been used as a mathematical tool to look at the non linear behavior of various mechanical and electrical systems [4]. Volterra series has been introduced. Two methods for determination of volterra kernels are specified and harmonic input method is used for analysis. Simulations for different order harmonics are done which represent varying degrees of non linearity.

Index Terms - Volterra series, Non-linear systems

1. INTRODUCTION

Virtually all physical systems are non linear in nature. Sometimes it is possible to describe the operation of a physical system by a linear model, if the operation of the physical system does not deviate too much from the normal set of operating conditions. But in analyzing the behavior of any physical system, one often encounters situations where linear models are inadequate or inaccurate, that is the time when concepts like Volterra series prove useful. Volterra series takes into account the non linear behavior of a system.

2. REPRESENTATION

Any time-invariant, nonlinear system can be modeled as an infinite sum of multidimensional convolution integrals of increasing order. This is represented symbolically by the series of integrals called Volterra kernels. [2]

$$y(t) = \sum_{n=1}^{\infty} \left(\frac{1}{n!}\right) \int_{-\infty}^{\infty} \left(d\tau_1\right) \ldots \int_{-\infty}^{\infty} \left(d\tau_n\right) h(\tau_1 \ldots \tau_n) \prod_{i=1}^{n} x(t-\tau_i) \qquad (2)$$

This series is known as the Volterra series. Here y(t) represents the system response. Each of the convolution integrals contains a kernel, either linear (h_1) or nonlinear (h_2,..,h_n), which represents the behavior of the system.

Volterra kernels, both linear and nonlinear, are input dependent. The first order kernel, h_1, represents the linear unit impulse response of the system. The second order kernel, h_2, is a two-dimensional function of time. It represents the response of the system to two separate unit impulses applied at two varying points in time. Similarly the other higher order kernels represent the response of the system to a combination of different signals at varying points of time.

3. IDENTIFICATION OF HARMONICS

Let the input to a system, with a first order kernel only, be $x(t) = e^{j\omega t}$

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The output y(t) will be calculated as follows:

$$y(t) = \int_{-\infty}^{\infty} h_1(\tau) e^{j\varpi(t-\tau)} d\tau = e^{j\varpi t} \int_{-\infty}^{\infty} h_1(\tau) e^{-j\varpi\tau} d\tau$$

$$= H_{1\varpi}(j\varpi) e^{j\varpi t} \qquad (3)$$

where $H_{1\varpi}(j\varpi) = h_1(\tau) e^{-j\varpi\tau} d\tau$

The complex number $H_1\omega$ ($j\omega$) by which the output phasor is multiplied is called the transfer function or the first order harmonic. Similarly, higher order harmonics can be calculated.

4. DETERMINATION OF VOLTERRA KERNELS

When we have an equation relating the input x(t) to the output y(t) then we can obtain the volterra kernels by two methods [1]:

1. Harmonic input method: used for determination of kernels in frequency domain.
2. Direct expansion method: used for determination of kernels in time domain

4.1. Harmonic Input Method

[1]When the input is

$$x(t) = e^{j\varpi_1 t} + e^{j\varpi_2 t} + \ldots e^{j\varpi_n t} \qquad [1]\ (4.1)$$

where $\omega_i = 2\pi f_i$, i =1,2,…,n and the ω_i are incommensurable, then

$H_n\omega$ ($j\omega_1$,…, ω_n)={coefficient of [$e^{j\varpi_1 t} + e^{j\varpi_2 t} + \ldots e^{j\varpi_n t}$]}

The complexity of this method increases rapidly with n. [2]

$H_n\omega$ ($j\omega_1$,…, ω_n) is the nth order harmonic.

4.2. Direct Expansion Method

In this method, the system equations are manipulated until they are brought into the form of a Volterra series, and the h_n are simply "read off" the representation. This method gives good results when the value of n is large [2].

5. SIMULATIONS

The steps followed in the simulation are:

1. The system is represented in the form of differential equations.
2. Its solution is expressed as a truncated Volterra series expansion as follows
   $y(t) = H_1[x(t)] + H_3[x(t)] + H_5[x(t)]$
3. Taking x(t) to be a sinusoidal input, we get

$$\begin{aligned}
y(t) = & \ A*\mathrm{Re}\{ H_{1\omega}(j\omega)\, e^{j\omega t} \} \\
& + 2(A/2)^3 [\mathrm{Re}\{ H_{3\omega}(j\omega, j\omega, j\omega)\, e^{j3\omega t} \}] \\
& + 2(A/2)^3 [\mathrm{Re}\{3 H_{3\omega}(j\omega, j\omega, -j\omega)\, e^{j\omega t} \}] \\
& + 2(A/2)^5 [\mathrm{Re}\{ H_{5\omega}(j\omega, j\omega, j\omega, j\omega, j\omega)\, e^{j5\omega t} \}] \\
& + 2(A/2)^5 [\mathrm{Re}\{ 5H_{5\omega}(j\omega, j\omega, j\omega, j\omega, -j\omega)\, e^{j3\omega t} \}] \\
& + 2(A/2)^5 [\mathrm{Re}\{ 10H_{5\omega}(j\omega, j\omega, j\omega, -j\omega, -j\omega)\, e^{j\omega t} \}] \qquad (5)
\end{aligned}$$

where $H_{1\omega}$, $H_{3\omega}$ and $H_{5\omega}$ for different arguments can be found out by harmonic input method for determination of kernels.

4. The values of these kernels are then substituted in the system equation to determine the linear and the non-linear part.

5.1. Volterra Analysis Of A Non-Linear Spring

The equation of a nonlinear spring is given by

$y(t) = mx''(t) - b[x'(t)]^3 + kx(t)$ [1] (5.1)

Applying the harmonic input method and taking x(t) as $e^{j\omega t}$, we get

$H_1[x(t)] = A*Re((-m\omega^2+k)e^{j\omega t})$

$H_3[x(t)] = 2(A/2)^3*(Re((-2m\omega^2+2k)e^{3j\omega t}) + Re(3(-2m\omega^2+2k-12bj\omega^3)e^{j\omega t}))$

$H_3[x(t)] + H_5[x(t)] = 2(A/2)^3*(Re((-2m^2+2k)e^{3j\omega t}) + Re(3(-2m\omega^2+2k-12bj\omega^3)e^{j\omega t})) + 2(A/2)^5*(Re(5(64bj\omega^3)e^{3j\omega t}) + Re(10(-3m\omega^2+2k-54bj\omega^3)e^{j\omega t}))$

[1] Using the steps described in the section 4, this problem was simulated for A=2, b=2, m=0.001kg, k=3 and $\omega=\pi$ and graphs were obtained as shown in the figure:

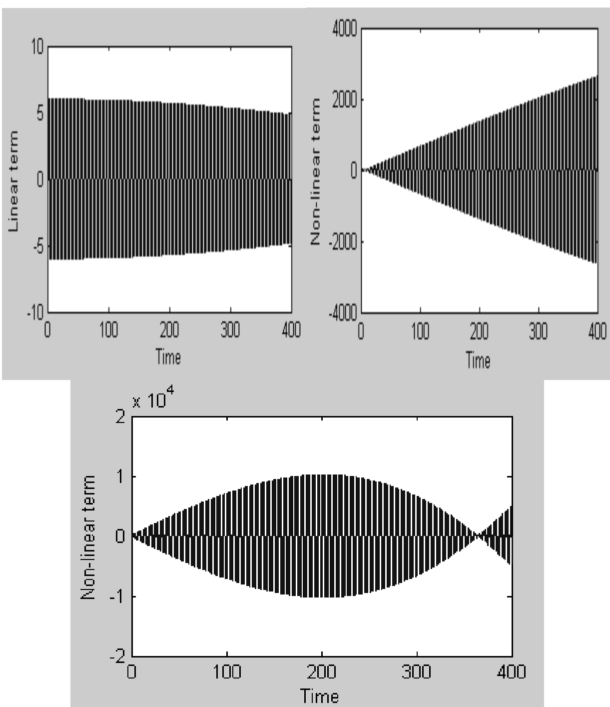


Figure 5.1: Simulation graphs for a Nonlinear Spring (Clockwise from top left) a) Linear part b) Third harmonic c) Third and fifth harmonic

5.2. Volterra Analysis of a Simple Pendulum

The equation of motion of a simple pendulum with linear damping is given by

$y(t) = x''(t) + ax'(t) + b\sin x(t)$ [3] (5.2)

Normally for any non linear system we consider $\sin x(t) \approx x(t)$ for small x(t). Here we consider Volterra systems to take into account the non-linearity caused by sine[1]

Using the steps described in the section 4, this problem was simulated for A=1, b=0.2, a=2, k=3, m=0.001 and $\omega=\pi/2$ and graphs were obtained as shown in the figure:

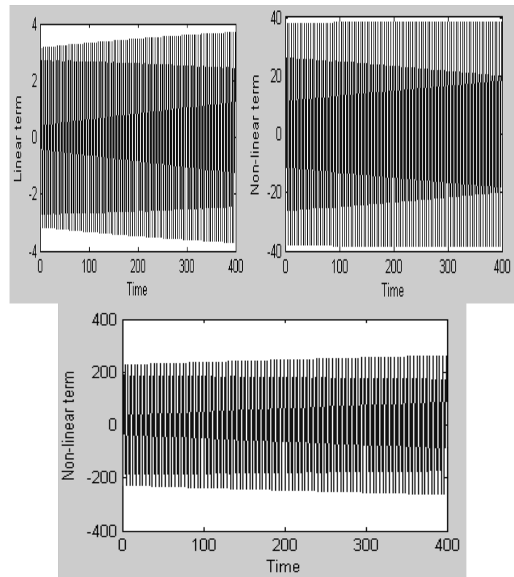


Figure 5.2: Simulation graphs for a simple pendulum (Clockwise from top left) a) Linear part b) Third harmonic c) Third and fifth harmonic

5.3. Volterra Analysis of a Lr Network

The differential equation for a LR network can be expressed as

$V(t) = Lq''(t) + Rq'(t)$ (5.3.1)

Where

V(t) is the voltage supplied

L is the inductor

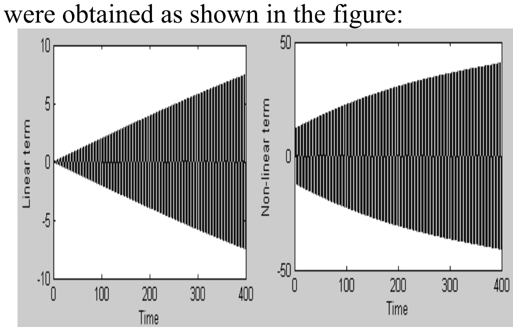
R is the resistor

q(t) is the charge

Using the force voltage analogy we get the following equation for a LR network

$y(t) = mx''(t) + b[x'(t)]^3$ (5.3.2)

Using the steps described in the section 4, this problem was simulated for A=2, b=2, m=0.001kg, k=3 and $\omega=\pi$ and graphs were obtained as shown in the figure:



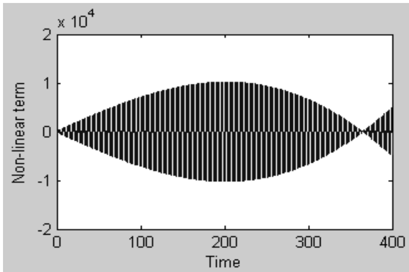


Figure 5.3: Simulation graphs for a LR network (Clockwise from top left) a) Linear part b) Third harmonic c) Third and fifth harmonic

5.4. Volterra Analysis of a Rc Network

The differential equation for a RC network can be expressed as

$V(t) = Rq'(t) + q(t)/C$ (5.4.1)

Using the force voltage analogy we get the following equation for a RC network

$y(t) = cx'(t) + kx(t) - b[x'(t)]^3$ (5.4.2)

Using the steps described in section 4, this problem was simulated for A=2, b=2, k=3, $c=2*10^5$ and $\omega=\pi$ and graphs were obtained as shown in the figure:

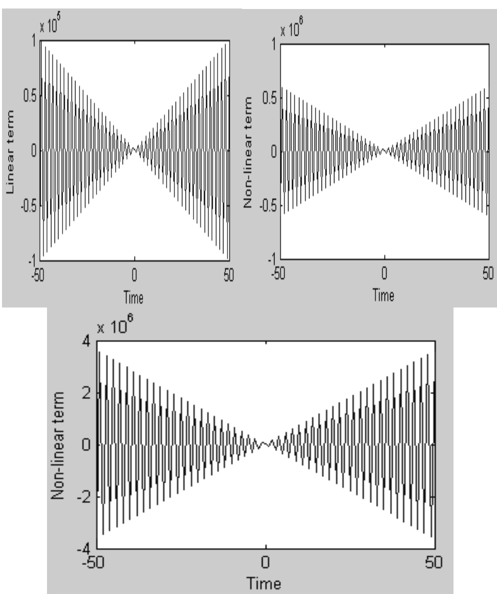


Figure 5.4: Simulation graphs for a RC network (Clockwise from top left) a) Linear part b) Third harmonic c) Third and fifth harmonic.

5.5. Volterra Analysis of A Rlc Network

The differential equation for a RLC network can be expressed as

$V(t) = Lq''(t) + Rq'(t) + q(t)/C$ (5.5.1)

RLC network can be realized by adding an external damper with the damping constant 'c' to the non linear spring system described above. This damper acts as the resistor.

The differential equation thus can be expressed as

$y(t) = mx''(t) - b[x'(t)]^3 + c\,x'(t) + kx(t)$ (5.5.2)

Applying the harmonic input method and taking x(t) as $e^{j\omega t}$, we get

$H_1[x(t)] = A*Re((cj\omega+k-m\omega^2)e^{3j\omega t})$

$H_3[x(t)] = 2(A/2)^3*(Re((27bj\omega^3)e^{3j\omega t}) + Re(3(2k-12bj\omega^3+2cj\omega)e^{j\omega t}))$

$H_3[x(t)] + H_5[x(t)] = 2(A/2)^3*(Re((27bj\omega^3)e^{3j\omega t}) + Re(3(2k-12bj\omega^3+2cj\omega)e^{j\omega t})) +2(A/2)^5*(Re(5(64bj\omega^3)e^{3j\omega t}) + Re(10(-3m\omega^2+3k-54bj\omega^3+3cj\omega)e^{j\omega t}))$

Using the steps described in section 4, this problem was simulated for A=2, b=2, m=0.001kg, k=3, $\omega=\pi$ and $c=2*10^5$ and graphs were obtained as shown in the figure:

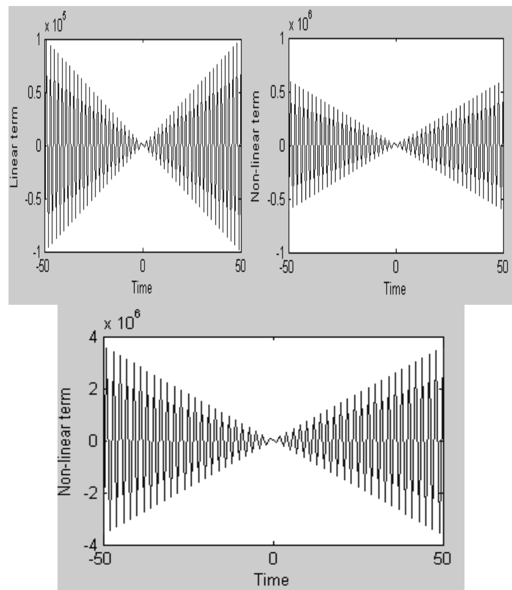


Figure 5.5: Simulation graphs for a RLC network (Clockwise from top left) a) Linear part b) Third harmonic c) Third and fifth harmonic.

6. APPLICATIONS OF VOLTERRA SERIES

The Volterra series finds application in a variety of fields ranging from medicine to system identification. It is widely used in biomedical engineering and neuroscience. It is used in electrical engineering to model intermodulation distortion in many devices including power amplifiers and frequency mixers. Its ability to provide closed form expressions for distortion components in terms of circuit parameters in analog circuits makes it an efficient method for analysis of distortion in such circuits [5].

General non-linear filters based on Volterra series are used for estimating signals corrupted by additive non Gaussian noise [6]. The series also finds use in nonparametric black-box modeling particularly for pharmacodynamics systems. These systems exploit the generality of higher order Volterra representations which can be used to describe and predict the response of an arbitrary pharmacokinetics or pharmacodynamics system without any prior knowledge on the structure of the system [7].

6. CONCLUSION

The paper discusses the application of Volterra series to non-linear mechanical as well as electrical circuits. The first, third and fifth harmonics of these non-linear systems have been simulated. The results of simulation of the differential equations of these systems show that the even order harmonics are zero while the odd order harmonics address the non linearity of the system. The results for higher order harmonics give a more accurate picture of the behavior of the system. These results show that incorporating the non-linearity of the system adds to the accuracy of system behavior representation.

FUTURE SCOPE

In an image-processing environment, it is known that linear filters are not able to remove the noise, in particular the impulsive one superimposed on a picture, without blurring the edges. Moreover, it is often necessary to take into account the intrinsic nonlinear behavior of the human visual system or of the optical imaging systems, resulting from the quadratic relation between the optical intensity and the optical field. For all these reasons, recently much attention has been drawn to the problem of nonlinear system modeling with a Volterra series expansion. Quadratic Volterra filters are used for such a purpose. Some other successful applications of volterra filters have been developed in system identification, signal processing, image processing, channel equalization, echo cancellation and telecommunication areas.

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