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Estimation of f-validity of Geometrical Objects with OWA Operator Weights

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Abstract - In the age of sophisticated crimes and terrorism, there is a requirement to develop a perception based multi criteria decision making system that can reveal the hidden clues in the environment of uncertainty. The crime has no fixed dimension to be carried out in. But, still there remain some imprecise clues for the crime site investigation team and precise interpretation of these clues is impossible. That is the place where role of proposed extended fuzzy logic (FLe) comes into play. Moreover, for decision making, crime site investigation team has to consider multi criteria with different weights under the uncertainty. So, Extended Fuzzy Logic and Ordered weighted Averaging (OWA) may be taken together as a double folded milestone in revealing the uncertainty in the world of computational forensic. The concept of unprecisiated fuzzy logic (Flu) was introduced by Zadeh. When a perfect solution cannot be given or process falls excessively costly then the role of concept of Flu comes into play. This novel concept provides the basis for FLe. In order to have a better understanding of Flu, the concept of fuzzy geometry (f-geometry) is introduced. The proposed work is based on Sketching with Word technique. We have introduced some fuzzy theorems (f-theorems) in proposed work. These f-theorems can be used for estimating the membership value of fuzzy objects in f-geometry. These f-objects may play vital role for identifying clues in computational forensic.

Index Terms – Fuzzy Theorem, Fuzzy Similarity, Fuzzy Validity, Fuzzy Geometry, OWA

1.0 INTRODUCTION

In computational forensics when exact interpretation of imprecise information at crime site is impossible e.g. clue left behind by criminal such as finger prints, shoe prints, and face-sketch drawn by experts on the basis of onlooker's statement, then role of extended fuzzy logic (FLe) [1][2] comes into play. Face-sketch drawn by experts is a matter of degree of perception rather than measurement. Sometimes forensic experts have to make decision on the basis of multiple criteria. The ordered weighted averaging (OWA) provides a unified decision making platform under the uncertainty [3]. The above said problem is attracting attention of researchers and scientists to merge the concept of extended fuzzy logic and OWA in forensic science. So, Extended Fuzzy Logic and OWA may be taken together as a double folded milestone in revealing the uncertainty in the world of computational forensics. Hence in this work the concept of Flu is used for the fuzzy proof (f-proof) of some complex f-theorems. Further, the results have improved by using the concept of OWA.

Fuzzy Logic, first introduced by L. A. Zadeh, provides a precise conceptual system of reasoning where information at hand is imperfect. When a perfect solution cannot be given or process falls excessively costly then the role of concept of unprecisiated fuzzy logic (Flu) comes into play. The concept of Flu was also introduced by Zadeh. This novel concept provides the basis for FLe. In order to have a better understanding of Flu, the concept of f-geometry is provided in literature [1][2][4][5]. The f-geometry is a counter part of Euclidian geometry in crisp theory. In f-geometry, figures are drawn by free hand. In Flu, there is concept of perception based fuzzy valid (f-valid) reasoning. In [4]-[6] authors apply novel approach for the estimation of perceptions in geometric shapes, which become the basis for the complex shape. For all the geometric shapes, the estimation of membership function is done on the basis of perception. The proposed work is based on sketching with words technique. In [7] authors have applied Yager's OWA weights for aggregating different components of f-objects. Whereas in [8], Minimizing Distance form Extreme Point OWA model is used for estimating the fuzzy validity (f-validity) of fuzzy Rhombus.

This paper is organized as follows. In Section 2, we have briefly looked into the related work. In section 3, we have discussed the basic f-objects of f-geometry followed by their estimation. The section 4 incorporates some of the existing f-theorems. In section 5, we have proposed some f-theorems and their respective f-proofs. The section 6 consists of the experimental work. The final section 7 comprises conclusion and future directions.

2.0 RELATED WORK

In [9], for fuzzy image processing membership values are assigned to some of the properties like brightness, edginess, homogeneity etc. of an image, in case of fuzziness and then in defuzzifying it to appropriate gray levels for computation. In the above work, application of fuzzy geometry is done with feature extraction, feature segmentation, and feature representation. There are number of factors which demarcates f-geometry from Poston's fuzzy geometry [10], coarse geometry [11], Rosenfeld fuzzy geometry [12], Buckley and Eslamis's fuzzy geometry [13], Mayburov's fuzzy geometry [14], and Tzafesta's fuzzy geometry [15]. The major one is that FLe allows f-valid reasoning based on perception in place of allowing p-valid reasoning which is based on measurement.

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In [16], OWA was applied to find the importance of weight for each of the document viewed by the user in web searching. With many aggregation methods introduced in fuzzy information processing tasks [17,18], the OWA operators are mainly described in detail in [3], has been applied in many applications including fuzzy logic controller [19], market analysis[20], image compression in [21]. In [22], environmental indices were developed using fuzzy numbers OWA operators, for finding similarity with multiple linguistic parameter as inputs. In [23], content based image retrieval from XML-based DBMS unites some features in an image; indexing structure uses Euclidean distance for individual feature is used. Moreover, ordered weighted averaging is used to aggregate the distance function of the features, support nearest neighbor and fuzzy queries. In [24], similarities among images are computed for retrieving similar images from the database that combines weighted averaging, Choquet Integral, and relevance feedback for a better performance. However, there has been much of work in image retrieval but with very little intelligence to recognize fuzzy objects and image. In [4],[7] and [8] OWA is applied for estimating f-validity of different f-objects.

3.0 FUZZY GEOMETRY

Flu introduces the concept of f-geometry. In Euclidean geometry crisp concept C corresponds to fuzzy concept f-C, in f-geometry. The fuzzy geometric shapes have the following elements.

3.1 fuzzy Points

In f-geometry, a point is said to be f-point, if radius is not exactly zero, but has haziness please refer to Fig 1(a). The membership function of f-point is shown in Fig. 1(b). The membership value of a f-point decreases with an increase in the length of radius d [4].

In equation (1) 'r' is the radius. From Fig.1(b) we can conclude that if the value of r is equal to 0 then we can conclude that given f-point is an exact point with validity index 1.

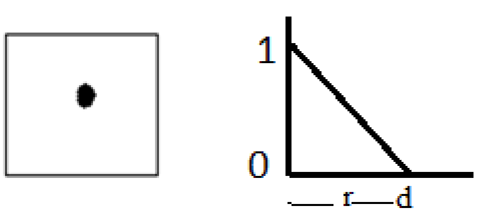

Figure 1: (a) f-point (b) Membership function

$$\mu(\text{f- point}) = \begin{cases} \frac{d-r}{d} & \text{if } 0 \leq r \leq d \\ 0 & \text{if } d \leq r \end{cases} \quad (1)$$

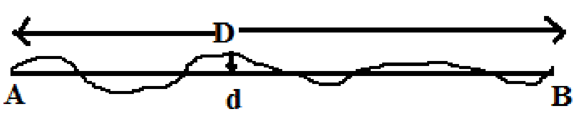

Figure 2: f-line

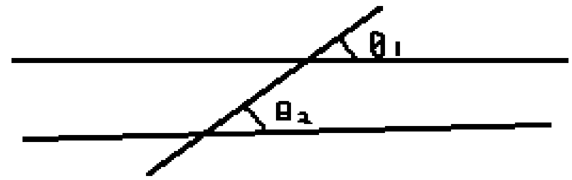

Figure 3: f-parallel

3.2 fuzzy Line

Let us consider an f-line as shown in Fig.2, which is like a curve that passes through a straight line *AB,* such that the distance between any point on the curve and the straight line *AB* is very small or negligible. With reference to Fig. 2, this implies that we are assigning a small value to the distance d, with the understanding that d is inferred as an imprecise value [4].

$$\mu(\text{f- line}) = \begin{cases} \frac{c-d}{c-b} & \text{if } b \leq d \leq c \\ 0 & \text{if } c < d \end{cases} \quad (2)$$

Equation (2) is membership function of f-line , where d << D, and 'd' is the maximum distance between the f-line and straight line and 'D' is the length of the straight line AB. Any line with little increase in difference from the straight line results in a decrease of the membership value.

3.3 fuzzy Parallel

In f-geometry, two lines are said to be f-parallel, if its membership value is closer to the membership value of parallel lines, and the membership function decreases with an increase in the difference of the corresponding angle θ as given by (3).

$$\mu(\text{f-similar angle}) = \begin{cases} \frac{c-h}{c-b} & \text{if } b \leq h \leq c \\ 0 & \text{if } c < h \end{cases} \quad (3)$$

h is the difference between angles θ_1 and θ_2.

Since the membership function decreases with the increasing value of h, we have estimated the membership function for two lines as f-parallel in (3).

3.4 fuzzy Triangle

In f-geometry a shape is said to be fuzzy triangle (*f*-triangle) if its membership value is closer to the membership value of triangle. As shown in Fig.4(b).

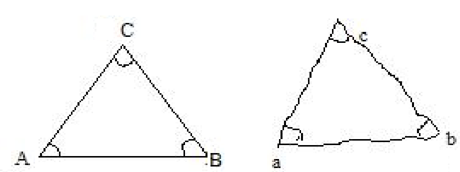

Figure 4: (a) Crisp-triangle (b) f-triangle

Figure 5: Estimating the *f*-interior angle of *f*-trianlge

The membership function of f-triangles is given by using some rules of geometry. Let us prove *f*-triangle by constructing it using (i) three *f*-lines, and (ii) three *f*-angles, as shown in Fig 5. We find the interior *f*-angle of an *f*-triangle by drawing f-altitude opposite to the angle.

Thereafter, we find the length of the base (x) and *f*-altitude (y) as shown in Fig. 5.

$$\theta_1=\tan^{-1}(y/x)$$

We substitute the values of x and y for estimating the *f*-interior angle θ_1 . The same method is followed for finding the other two *f*-interior angles θ_2 and θ_3.
In (4) μ_{SIA} denotes the membership value of sum of internal angles, where θ is given by $\theta=\theta_1+\theta_2+\theta_3$.

$$\mu(\text{f- SIA}) = \begin{cases} \dfrac{\theta-a}{b-a} & \text{if } a\le\theta\le b \\ \dfrac{c-\theta}{c-b} & \text{if } a\le\theta\le b \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

Where θ, a, and b are real numbers in equation 4.
The membership of f-triangle is given by (5).

$$\mu\,(\text{f-triangle})=\mu_{d1}*\mu_{d2}*\mu_{d3}*\mu_{SIA} \qquad (5)$$

Here μ_{d1}, μ_{d2}, and μ_{d3} denote the membership values of f-line with distance d_1, d_2, and d_3 from the reference straight line given by (2) respectively. The μ_{SIA} denotes the membership value of sum of internal angles given by equation (4).

4.0 F-Theorem

The f-theorem in f-geometry is f-transform of a theorem in Euclidean geometry. In f-theorem, we are trying to formalize the f-concept in f-geometry, generally in the form of membership functions, e.g. by transforming of some rules of crisp geometry into f-geometry.

4.1 f-similarity and f-validity

In f-geometry any two f-objects are said to be f-similar, if both of them have same shape. Very specifically, by uniform scaling one must be congruent to other. Conversely, f-similar polygons may be of same f-angles and scaling of f-sides may be proportionate. This section illustrates the concept of f-similarity by using well known triangle postulates. There are three well known postulates. All the interior angles are same (AAA), all the sides are same in proportion (SSS), and two sides are same in proportion with a same angle (SAS). These foresaid postulates prove triangles to be similar. Assuming a formal illustration of the concept of the f-theorem, let us consider that ABC constitutes a triangle with three straight lines and three interior angles, as shown in Fig. 4(b).

4.1.1 Angle Angle Angle (AAA): In f-geometry, two triangles are said to be f-similar if their membership function has high validity index to the property of similar triangles (AAA) and the membership values decrease with the increase of difference of the corresponding angles. The high validity index in this context refers to the degree of closeness. It is mathematically represented as

$$\mu(\text{f-similar})=\mu_{A1}*\mu_{A2}*\mu_{A3} \qquad (6)$$

Where μ_{A1}, μ_{A2}, and μ_{A3} are membership of angle1, angle2, angle3 are based on the difference in the corresponding angles.

4.1.2 Side Side Side (SSS): In f-geometry, two triangles are said to be f-similar if their membership function has high validity index to the property of similar triangles (SSS), with all the three corresponding sides are equal in f-proportion and the membership values decrease even in slight difference in proportion of the sides. Mathematically represented as

$$\mu(\text{f-similar})=\mu_{S1}*\mu_{S2}*\mu_{S3} \qquad (7)$$

Where μ_{S1} , μ_{S2} , and μ_{S3} are memberships of f-proportions of corresponding side1, side2, side3 respectively.

4.1.3 Side Angle Side (SAS): In f-geometry, two triangles are said to be f-similar if their membership function has high validity index to the property of similar triangles (SAS) and the membership values decreases in difference in corresponding angle and difference in proportion of two corresponding sides. Mathematically represented as

$$\mu(\text{f-similar})=\mu_{S1}*\mu_{A2}*\mu_{S3} \qquad (8)$$

Where μ_{S1}, μ_{A2}, and μ_{S3} membership functions of side1, angle2, side3 respectively. In case of SAS, we assume that AB/DC' *=BC/AD' *= k (A constant) i.e. corresponding sides of the two triangles are in the same ratio as in geometry. Here AB/DC' and BC/AD' takes the fuzzy proportion values $k_1$and k_2 respectively. Point to be noted here is AB/DC' *= BC/AD means AB/DC' is approximately equals to BC/AD [4-6]. The membership function of the f-similar side and f-similar angle are computed by (9) and (10) respectively.

$$\mu(\text{f-similar side})= \begin{cases} \dfrac{c-j}{c-b} & \text{if } b\le j\le c \\ 0 & \text{if } c\le j \end{cases} \qquad (9)$$

Where j is given by k-k1 and k-k2 .

$$\mu(\text{f-similar angle})= \begin{cases} \dfrac{c-l}{c-b} & \text{if } b\le l\le c \\ 0 & \text{if } c\le l \end{cases} \qquad (10)$$

Here $l=\theta_1-\theta_2$ is the difference between angles.

The f-similarity of triangles is given by

$$\mu_{SAS}\text{(f- similarity)}=\text{f-val1}*\text{f-val2}*\mu_{diff1}*\mu_{diff2}*\mu_{diff3} \quad (11)$$

Where f-val1 and f-val2 are the f-validities of corresponding triangles and given by (5). The μ_{diff1} and μ_{diff2} are membership of the difference between corresponding f-interior angles is calculated by (10).

5.0 PROPOSED F-Theorem

In this section we have introduced the definition of fuzzy parallelogram (f-parallelogram) and fuzzy rhombus (f-rhombus). Further we have discussed f-theorems and their f-proof for f-parallelogram and f-rhombus by using validation principle. By using the proposed methodology we can introduce more complex f-objects. These f-objects may play vital role for identifying clues in computational forensic**.** This section further comprises of some proposed f-theorems and their f-proofs. In f-geometry, f-proof may be either empirical or logical. The empirical f-proof involves experiments while the logical f-proof is the f-transform of their counterpart of Euclidean geometry. An important principle in f-geometry is validation principle, "Let *p* be a *p*-valid conclusion drawn from a chain of premises $p^1,\ldots,p^n$. Then, using the star notation,$*p$ is an *f*-valid conclusion drawn from$*p^1,\ldots,*p^n$ and $*p$ has a high validity index. It is this principle that is employed to derive *f*-valid conclusions from a collection of *f*-premises"[1].The validation principle leads to the following assertion in *f*-geometry. If ABC and *ACD* are *f*-similar *f*-triangles then ABCD be f-parallelogram. In f-theorem 1 to 4 Fig. 6 is taken as an example. In theorem 5 Fig. 7 is taken as an example.

5.1 f-parallelogram

A shape ABCD as shown in Fig.6, is called f-parallelogram, if both pairs of opposite f-sides are f- parallelogram and represented in terms of membership function as [7][8]

$$\mu(\text{f-p})=\mu_D*\mu_{diff} \quad (12)$$

$$\mu(\text{f-p})=\mu_{d1}*\mu_{d2}*\mu_{d3}*\mu_{d4}*\mu_{diff1}*\mu_{diff2} \quad (13)$$

Where μ_{d1},μ_{d2},μ_{d3}, and μ_{d4} are the individual membership values of f-Sides AB, BC, CD, and AD respectively as given by (2). The μ_{diff1} and μ_{diff2} are the membership values of difference of the corresponding angles given by (10) .

5.2 f-rhombus

In a shape (as shown in Fig. 7) if all of its f-sides are f-similar such that the difference between all four sides are very small or negligible, then it is called f-rhombus [7][8]. The f-rhombus is represented by (14) in terms of membership function.

$$\mu(\text{f-rhombus})=\mu_{d1}*\mu_{d2}*\mu_{d3}*\mu_{d4} \quad (14)$$

Where μ_{d1},μ_{d2},μ_{d3}, and μ_{d4} are the individual membership values given by (9).

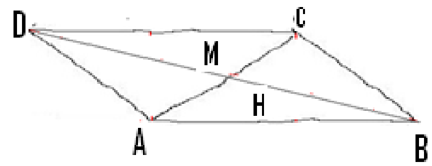

Figure 6: f-parallelogram

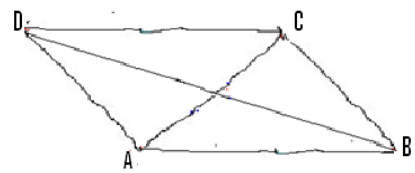

Figure 7: f-rhombus

5.3 f-theorems

5.3.1 f-theorem1: If ABCD be a f-parallelogram with a diagonal f-line AC. Then f-line AC divides f-parallelogram into two f-similar triangles ABC and ACD.

Above f-theorem has been derived by using validation principle. Let ABCD be the f-quadrilateral and consist of two f-triangles ABC and ACD. If f-triangles ABC and ACD are f-similar then ABCD will be the f-parallelogram. Higher degree of similarity of f-triangles ABC and ACD leads to f-valid conclusion, "The quadrilateral ABCD has higher degree of validity index of f-parallelogram".

5.3.1.1 f-proof : The f-similarity of f- triangles ABC and ACD is calculated by (11).

$$\mu_{SAS}\text{(f-similarity)} =\text{f-val}_{ABC} * \text{f-val}_{ACD}*\mu_{diff1}*\mu_{diff2}*\mu_{diff3} \quad (15)$$

Where f-val_{ABC} and f-val_{ACD} are the f-validities of triangle ABC and ACD respectively. The μ_{diff1} and μ_{diff2} are the differences of the corresponding sides given by (9). The μ_{diff3} is difference of the corresponding f-interior angle which is calculated by (10). The f-val1 and f-val2 are evaluated by (5) as follows.

$$\mu(\text{f-validity1})=\mu_{d1}*\mu_{d2}*\mu_{d3}*\mu_{SIA}$$

Where μ_{d1},μ_{d2}, and μ_{d3} denote the membership values of f-sides AB, BC, and CD with distance d_1, d_2, and $d_3$from the reference straight line respectively. The μ_{SIA} denotes the membership value of sum of internal angles.

$$\Sigma\theta= \angle CAB+ \angle ABC+\angle ACB,$$

Where $\angle$ CAB, $\angle$ ABC, and $\angle$ ACB are the internal angles of triangle ABC.

$$\mu(\text{f-validity2})=\mu_{d1}*\mu_{d2}*\mu_{d3}*\mu_{SIA}$$

Where μ_{d1},μ_{d2}, and μ_{d3} denote the membership values of f-line AC, AD, and DC with distance d_1, d_2, and $d_3$from the reference straight line respectively. The μ_{SIA} denotes the membership value of sum of internal angles

$$\Sigma\theta = \angle CAD+\angle ADC+\angle ACD$$

Where $\angle$ CAD, $\angle$ ADC, and $\angle$ ACD are the internal angles of triangle ADC.

The membership function of each f-line and sum of interior angles are calculated by using (4) and (5) respectively.

5.3.2 f-theorem2: In f-quadrilateral, if difference of each pair of opposite f-angle is either very small or negligible then ABCD is f-parallelogram..

5.3.2.1 f-proof : The quadrilateral ABCD has higher degree of validity index of f-parallelogram if the value of $\delta\theta_1$ and $\delta\theta_2$ are small or negligible .

Where $\delta\theta_1$ and $\delta\theta_2$ are given by $\delta\theta_1 = \angle DCB - \angle DAB$

$$\delta\theta_2 = \angle ADC - \angle ABC$$

The validity index is given by (16)

$$\mu(\text{f-validity}) = \mu_{diff1} * \mu_{diff2} * \mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4} \quad (16)$$

The μ_{diff1} and μ_{diff2} denote the membership values of difference of opposite angles $\delta\theta_1$ and $\delta\theta_2$ respectively. The value of μ_{diff1} and μ_{diff2} is evaluated by (3). Where μ_{d1}, μ_{d2}, μ_{d3}, and μ_{d4} are memberships of f- lines AB, BC, DC, and AD with distance d_1, d_2, d_3, and d_4 from the reference straight line respectively.

5.3.3 f-theorem3: If the diagonals (f-line) of f-quadrilateral bisect each other then it will be a f-parallelogram.

5.3.3.1 f-proof: The quadrilateral ABCD has higher degree of validity index of f-parallelogram if the value of distance *diff* between intersection point H of diagonals AC and BD from midpoint of any diagonal is either small or negligible .

$$diff = H - M$$

Where validity index given by

$$\mu(\text{f-validity}) = \mu_{diff1} * \mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4} * \mu_{d5} * \mu_{d6} \quad (17)$$

The value of μ_{diff} denotes the membership value of the distance given by (2). The μ_{d1}, μ_{d2}, μ_{d3}, and μ_{d4} are membership values of f-lines AB, BC, CD, and AD respectively. The μ_{d5} and μ_{d6} denotes the membership values of f-lines AC and DB with distance d_5 and d_6 from the reference straight line respectively.

5.3.4 f-theorem4: A f-quadrilateral is f-parallelogram if a pair of opposite sides is f-equal and f-parallel.

5.3.4.1 f-proof : The quadrilateral ABCD has higher degree of validity index of f-parallelogram if the difference between opposite f-sides AB/CD and BC/AD and difference between opposite angle are either small or negligible. The validity index is given by (18).

$$\mu(\text{f-validity}) = \mu_{diff1} * \mu_{diff2} * \mu_{diff3} * \mu_{diff4} * \mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4} \quad (18)$$

Where μ_{diff1} and μ_{diff2} are denoting the membership value of f-similar side by (9) . μ_{diff3} and μ_{diff4} denoting the membership value of f-parallel line by (3). μ_{d1}, μ_{d2}, μ_{d3}, and μ_{d4} are memberships of f-line AB, BC, DC and AD with distance d_1, d_2, d_3, and d_4 from the reference straight line respectively.

5.3.5 f-theorem5*:* If diagonals (f-lines) of f-quadrilateral are perpendicular to each other then f-quadrilateral will be a f-rhombus.

5.3.5.1 f-proof*:* The f-quadrilateral ABCD has higher degree of validity index of f-rhombus, if the difference between interior angle made by diagonals from right angle is either small or negligible .

The validity index is given as

$$\mu(\text{f-validity}) = \mu_{IA1} * \mu_{IA2} * \mu_{d1} * \mu_{d2} \quad (19)$$

Here μ_{IA1} and μ_{IA2} are the individual membership values of interior angle1 and interior angle2 from the right angle respectively. μ_{d1} and μ_{d2} denote the membership values of diagonal f-lines AC and DB with distances d_1 and d_2 respectively.

The f-similarity and f-validity of f-objects is estimated by using multiplication and OWA methods. It is shown in results OWA method has significant improvement over multiplication method.

5.4 Simple Multiplication Method

This method is multiplication of all the membership values.

5.5 Ordered Weighted Averaging Method

Ordered Weighted Averaging (OWA) is the central concept of information aggregation, originally introduced by Yager[3]. OWA facilitates the means of aggregation in solving of problems that arises in multi criteria decision making. Furthermore, OWA operator provides a parameterized family of aggregation operators, including well-known operators such as maximum, minimum, arithmetic mean, k-order statistics, and median. Sometimes, exact **AND**-ness is required for multi-criteria decision making, which offers minimum value and sometimes exact **OR**-ness which offers maximum value. The OWA aggregation operator lies somewhere in between the two extremes of **AND**-ness and **OR**-ness. Two extremes are restricted to mutually exclusive probabilities for multiplication (like **AND**-gate) and summation (like **OR**- gate). Subsequent part discloses a brief account of OWA operators, detail discussion about the behavior of operators is in [3]. The OWA operation involves three following steps.

1) Reordering of inputs, 2) Weight determination related with OWA operators, and 3) Aggregation process.

Definition: "Mapping the OWA operator R from $R^m \rightarrow R$, (where R = [0, 1]), with dimension m, has weighting vector w= $(w_1, w_2, w_3, \ldots w_m)^T$, where $w_j \in [0, 1]$ and $\Sigma w_j = 1$, the summation of individual weights will always found to be one"[3]. Thus, for the input parameter $(x_1, x_2, x_3 \ldots\ldots x_m)$, the size of multicriteria will be m,

In vector $(y_1, y_2, y_3 \ldots . y_m)$ the y_j is the j^{th} largest number in the vector $(x_1, x_2, x_3, \ldots x_n)$, and $y_1 \geq y_2 \geq y_3 \geq \ldots \geq y_m$. However, the weights wj of the operator R are not related with any exact value of xj, instead they are related with the ordinal position of y_j [3]. The minimum and maximum range of values can be decided based upon the concept of **OR**-ness (β), as defined by (21) [3].

$$OWA(x_1, x_2, x_3 \ldots\ldots x_m) = \sum^{m} w_j y_j \qquad (20)$$

$$\beta = \frac{1}{m-1} \sum_{j=1}^{m} w_j (m-1) \qquad (21)$$

Here, β (***OR**-ness)* ranges between [0, 1]. On every occasion the value of β = 1, generates the weight vector as (1, 0, 0,...0). Thus, the maximum value of x_j acquires the entire weight, resulting the OWA operator as *maximum* operator. On the other hand, if β = 0, generates the weight vector as (0,0,0,.., 1). Thus, the minimum value of x_j will acquires the entire weight, resulting the OWA operator as *minimum* operator. When β = 0.5, generates the weight vector as (1/n, 1/n, 1/n,…, 1/n), means that arithmetic mean of weights are evenly distributed among the inputs. The membership function of a relative quantifier can be represented as

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$$Q(r) = \begin{cases} 0 & \text{if } r < a \\ \frac{r-a}{b-a} & \text{if } b \le r \le a \\ 1 & \text{if } r > b \end{cases} \qquad (22)$$

Where a, b, and r∈[0,1].

In [3], Yager calculates the weights w_j of the OWA aggregation from the function *Q* describing the quantifier, with m number of criteria.

$$w_j = Q(j/m) - Q((j-1)/m) \qquad (23)$$

6. EXPERIMENTAL WORK AND RESULTS

In this section the f-validity of f-parallelogram and f-rhombus is computed. The f-theorem1, f-theorem2, f-theorem3, f-theorem4, and f-theorem5 are illustrated in Example1, Example2, Exmaple3, Exmaple4, and Exmaple5 respectively. In Fig. 9 to 13 results of practical work is shown. The sample images which are taking inputs are shown in Fig.8.

Examples

Example1: The f-parallelogram shown in Fig.6 is constituted by two triangles ABC and ACD. The ABC has f-transformation distances for three f-lines AB, BC, and AC are 5, 49, and 9. This results in μ_d as {0.97, 0.74, 0.95}. The sum of interior angles is 177.16° which in turn results in μ_{SIA} as {0.716}.The *f*-validity is calculated by taking the product of the above membership values {0.97, 0.74, 0.95, 0.716}. The result comes to be 0.4882. For triangle ACD the f-transformation distances for three sides (f-lines) AC, DC, and AD is 9, 6, and 13. This results in μ_d as {0.95, 0.96, 0.93}. The sum of interior angles is 176° which in turn results in μ_{SIA} as {0.6}.The *f*-validity is calculated by taking the product of the above membership values {0.97, 0.74, 0.95, 0.6}. The result comes to be 0.513. The difference among the corresponding f-interior angles is found as {7} generates the membership values {0.3}.

The differences in proportion of corresponding f-sides are 0.02 and 0.03 generate membership values 0.98 and 0.97. To compute the f-similarity, we go for implementing the above data set by using (15). Then, the f-similarity is computed with the following values as:

$$\mu_{SAS}(\text{f-similarity}) = \text{f-val}_{ABC} * \text{f-val}_{ACD} * \mu_{diff1} * \mu_{diff2} * \mu_{diff3}$$

$$\mu_{SAS}(\text{f-similarity}) = \{\mu_{AB} * \mu_{BC} * \mu_{AC} * \mu_{SIA1} * \mu_{AC} * \mu_{DC} * \mu_{AD} * \mu_{SIA2} * \mu_{diff1} * \mu_{diff2} * \mu_{diff3}\}$$

$$\mu_{SAS}(\text{f-similarity}) = 0.0714$$

The quadrilateral has 0.0714 validity index of f-parallelogram. In OWA method, we have considered all 11 important parameters by using (15) as inputs, i.e. the size of input vector m = 11.The fuzziness in f-parallelogram is computed using the OWA operator R for the linguistic quantifier "most" i.e. a=0.3 and b= 0.8 by (22). The weight vector generated by (23) is (0,0,0,0.1273, 0.181818, 0.181818, 0.181818, 0.181818, 0.145455) used in (20) with membership values 0.3,0.6, 0.716, 0.74, 0.74, 0.93, 0.95,0.95,0.9,0.97 and 0.98 produces f-similarity of 0.8 which is a significant improvement over 0.07 produced by multiplication method. The corresponding results are shown in Fig. 9.

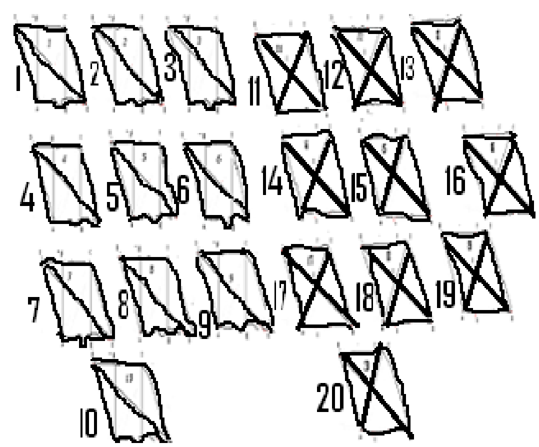

Figure 8: Sample Images

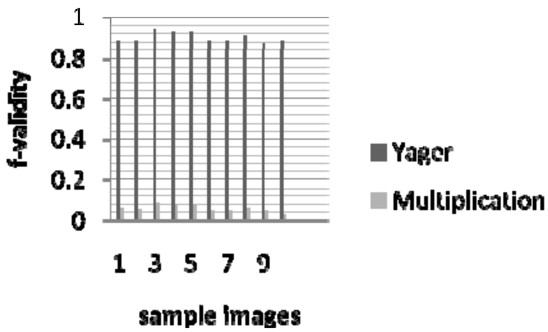

Figure 9: Comparision of f-validity generated by Yager and simple multiplication method by theorem 1

Example 2: In f-parallelogram shown in Fig.6, f-transformation distance for the f-lines AB, BC, CD and AD are 5, 49, 6, and 13. This results in μ_d as {0.97, 0.74, 0.96, 0.93}. The differences of the opposite interior angles $\angle DCB$- $\angle DAB$ and $\angle ADC$- $\angle ABC$ are 0.2 and 0.6. Which in turn results μ_{diff} as {0.98, 0.94}.The values of angle $\angle DCB$, $\angle DAB$, $\angle ADC$, and $\angle ABC$ are 56.4^0, 56.8^0, 123.6^0 and 123^0 respectively.

The f-validity is given by (16).

$$\mu(\text{f-validity})=\mu_{diff1} * \mu_{diff1} * \mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4}$$

$$\mu(\text{f-validity})=0.98*0.94*0.97*0.74*0.9681*0.9309$$

$$\mu(\text{f-validity})=0.595$$

In OWA method, we consider 6 important parameters by using (16) as inputs, i.e. the input value m = 6.The fuzziness in f-parallelogram will be computed using the OWA operator R for the linguistic quantifier "most" i.e. a=0.3 and b= 0.8 by (22). The weight vector generated by (23) is (0, 0, 0.333, 0.166, 0.166, 0.333, 0).The ordered membership values are (0.98, 0.97, 0.9681, 0.94, 0.9309,0.74).Produces the f-validity 0.953414 by (20).The significant improvement over multiplication method in the result can be seen in Fig. 10.

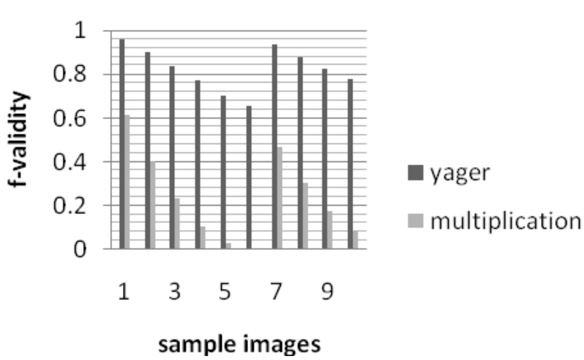

Figure 10: Comparision of f-validity generated by Yager and simple multiplication method by theorem 2

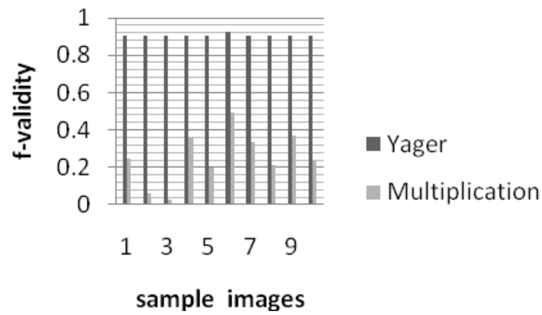

Figure 11: Comparision of f-validity generated by Yager and simple multiplication method by theorem 3

Example 3: In f-parallelogram shown in Fig.6, f-transformation distance for the f-lines AB, BC, CD, and AD are 5, 49, 6, and 13. This results in μ_d as {0.97, 0.74, 0.9681, 0.9309 }. The f-transformation distances of intersection points of diagonal AC and BD are 5 and 49.This results in μ_d as {0.97, 0.74}.

$$\mu(\text{f-validity})=\mu_{diff1} * \mu_{d1} * \mu_{d2} * \mu_{d3} * \mu_{d4} * \mu_{d5} * \mu_{d6}$$

The value of difference is 11.5 which generate μ_{diff} 0.42. Then, the f-validity is computed by (17).

$$\mu(\text{f-validity})=0.42* 0.97* 0.74*0.9681* 0.9309*0.97*0.74$$

$$\mu(\text{f-validity})=0.247454$$

To compute the level of f-validity by OWA in f-parallelogram, here we consider 7 important parameters by (19) as inputs, i.e., with *m* = 7, the fuzziness in an f-parallelogram will be computed using the OWA operator R for linguistic quantifier "most" a=0.3 and b= 0.8.The weight vector is (0,0,0.2571,0.2857,0.2857,0.1714,0) with membership values (1,0.95,0.93,0.640) produces f-validity 0.9072 over 0.247454. Please refer to Fig.11 which is showing the results after applying theorem 3 on sample images of Fig.8.

Example4: In f-parallelogram shown in Fig.6, f-transformation distance for the f-lines AB, BC, CD and AD are 5, 49, 6, and 13. This results in μ_d as {0.97, 0.74, 0.9681, 0.9309}. The product of membership values of μ_d is 0.646259.The proportion of opposite f-lines CD and AB is 1.05757566.The proportion of f-lines BC and AD is 0.849276. Then, the f-validity is computed by (18)

$$\mu(\text{f-validity})=\mu_{diff1} * \mu_{diff2} * \mu_{diff3} * \mu_{diff4} * \mu_{d1} * \mu_{d22} * \mu_{d3} * \mu_{d4}$$

$$\mu(\text{f-validity})=0.367$$

To compute the level of f-validity by OWA in f-parallelogram, 8 important parameters has been considered as inputs, i.e., with *m* = 8. The fuzziness in an f-parallelogram will be computed by using (20). The OWA operator R for linguistic quantifier "most" a=0.3 and b= 0.8. The weight vector is (0,0, 0..2571,0.2857,0.2857,0.1667 ,0.1667, 0.3333,0) with the membership values (0.97, 0.97,0.74, 0.9681, 0.93, 0.894,0.915)

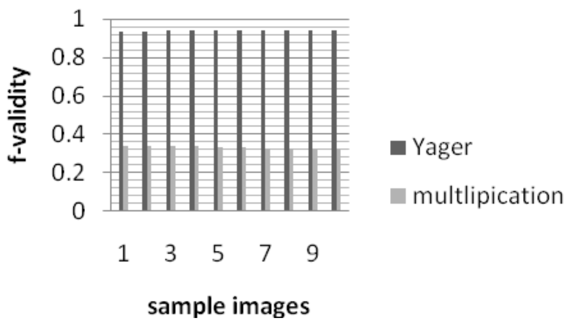

Figure 12: Comparision of f-validity generated by Yager and simple mutiplication method by theorem 4

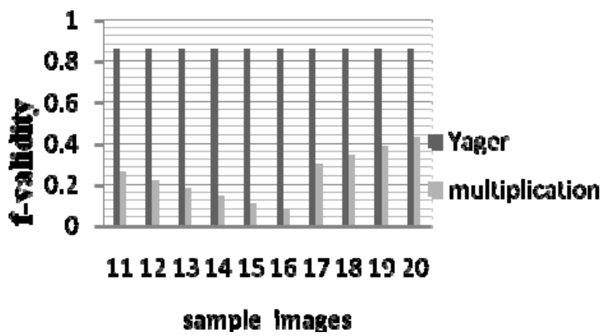

Figure 13: Comparision of f-validity generated by Yager and simple multiplication method by theorem 5

produces f-validity 0.9026.The experimental results are shown in Fig.12.

Example 5: In f-rhombus shown in Fig. 7, f-transformation distance of f-lines AC and DB are 9 and 13 respectively. The membership values of f-lines AC and DB are 0.95 and 0.93. The values of internal angles DEF and CEF are 102.254, 94.90173 respectively. Which in turn results μ as {0.38, 0.75}.Then, the f-validity is computed by (19)

μ (f-validity)=$\mu_{IA1}*\mu_{IA2}*\mu_{d1}*\mu_{d2}$= 0.258.

To compute the level of f-validity by OWA in f-parallelogram, here we consider 4 important parameters by (19) as inputs, i.e., with m =4, the fuzziness in an f-parallelogram will be computed using the OWA operator R for linguistic quantifier "most" a=0.3 and b= 0.8.The f-validity is 0.86.The weight vector generated by (23) is (0.0,0.4,0.5,0.1).The ordered membership values (0.38,0.75,0.93,0.95) produces f-validity 0.862.Comparison of results of Yager's and simple multiplication method is shown in Fig. 13.

7. CONCLUSION AND FUTURE DIRECTIONS

In this paper, we have given the f-definitions and f-theorems of f-geometry. Afterwards, we have estimated the f-similarity of f-geometric objects by using membership values, such as f-parallelogram and f-rhombus by using f-theorems. By using the proposed methodology we can introduce more f-objects. These f-objects may play vital role for identifying clues in computational forensic. The OWA operators are employed to aggregate the membership values of individual feature and produced more improved result. In future, the investigation will be on for new and more effective multi criteria decision making methods.

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