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A Novel Hénon Map Based Adaptive PSO for Wavelet Shrinkage Image Denoising

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Abstract - Degradation of images due to noise has led to the formulation of various techniques for image restoration. Wavelet shrinkage image denoising being one such technique has been improved over the years by using Particle Swarm Optimization (PSO) and its variants for optimization of the wavelet parameters. However, the use of PSO has been rendered ineffective due to premature convergence and failure to maintain good population diversity. This paper proposes a Hénon map based adaptive PSO (HAPSO) for wavelet shrinkage image denoising. While significantly improving the population diversity of the particles, it also increases the convergence rate and thereby the precision of the denoising technique. The proposed PSO uses adaptive cognitive and social components and adaptive inertia weight factors. The Hénon map sequence is applied to the control parameters instead of random variables, which introduces ergodicity and stochastic property in the PSO. This results in a more improved global convergence as compared to the traditional PSO and classical thresholding techniques. Simulation results and comparisons with the standard approaches show the effectiveness of the proposed algorithm.

Index Terms - HAPSO, Hénon Map, Image Denoising, Premature Convergence, Wavelet Thresholding.

1. INTRODUCTION

Digital images have become indispensable, and with it the process of removing noise from an image while retaining its significant features has become an important requirement for analyzing images, especially in the field of digital image processing.

Ever since the work of Donoho [1]-[3] on wavelet thresholding, researchers have been trying to come up with techniques to make the thresholding process more adaptive. SureShrink, a wavelet Shrinkage technique is one such technique. Particle Swarm Optimization (PSO), a stochastic population based metaheuristic inspired from swarm intelligence, can be used to optimize the entire process of wavelet shrinkage image denoising. However, conventional PSO may get stuck in the local optima due to its fast convergence rate and even decrease the population diversity. To avoid this premature convergence and improve the population diversity, a hénon map based adaptive PSO (HAPSO) approach is proposed. Here all the deciding parameters of PSO, the inertia weight, the cognitive and social components, and the control parameters are made adaptive by special means. The use of Hénon map chaotic sequences for control parameters in PSO helps in escaping from the local minima. Hence it introduces ergodicity, irregularity and stochastic property in the PSO to improve the global convergence. The Adaptive PSO (HAPSO) is then used for selecting the optimum values for the parameters: wavelet threshold, type of wavelet basis and the level of decomposition; to denoise the digital image. In this paper, after providing the necessary background theory for classical wavelet shrinkage denoising technique and standard PSO, we give a detailed description of the proposed HAPSO technique.

The principal objective is to compare the performance of HAPSO, with standard PSO based wavelet shrinkage denoising technique and classical wavelet shrinkage denoising techniques for effective image restoration.

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2. LITERATURE SURVEY

In recent years much advancement has been made in further optimizing the process of wavelet thresholding image denosing by using population based metaheuristics like Particle Swarm Optimization. Recently, a PSO-based approach was proposed by G. G. Bhatuda et al. [5] for learning the parameters of sub-band adaptive thresholding function for image denoising. In a different approach, modified Chaotic Particle Swarm Optimization was proposed by Xuejie Wang et al. [7] which used a chaotic PSO approach to optimize the wavelet threshold values. While to explore a complete solution space for suitable threshold, PSOshrink was proposed by Chan-Cheng Liu et al. [20]. Another variant of PSO, a Fast Particle Swarm Optimization was proposed by Du-Jin Liu et al. [9], for obtaining the most optimum wavelet threshold values. The earlier approaches, though generating high visual quality of images, have been more focused on optimizing a single wavelet parameter, the threshold values. In this paper, the HAPSO algorithm is proposed which uses an adaptive, ergodic version of PSO to optimize the wavelet threshold values, optimally select which wavelet basis to use from a set of mother wavelets and also, which level of decomposition to perform. More promising results are shown in terms of both PSNR and visual image quality as all the otherwise random parameters of PSO have been made adaptive in the proposed method.

3. WAVELET THRESHOLDING AND DENOISING

Wavelet thresholding is a nonlinear technique wherein an image or the given data is decomposed into wavelet coefficients. These detailed coefficients are then compared with a given threshold value, coefficients smaller than the threshold are set to zero while the others are retained or modified depending on the threshold rule. The image is then reconstructed from the modified coefficients, which is called *Inverse Discrete Wavelet Transform (IDWT)* [4]. Wavelet shrinkage denoising involves the following steps:

1. Acquire a noisy digital signal.
2. Compute a linear forward discrete wavelet transform of the noisy signal.
3. Perform a non-linear thresholding operation on the wavelet coefficients of the noisy signal.
4. Compute the linear inverse wavelet transform of the threshold wavelet coefficients.

Image denoising can be achieved by the following threshold techniques: VisuShrink[2] and SureShrink[3].

VisuShrink: Also known as Universal Shrinkage technique, this method follows the hard thresholding rule. The threshold value *th* is given as:

$$th = \sigma * \sqrt{2 * \log n} \tag{1}$$

where, σ^2 is the noise variance present in the signal and n represents the signal size or number of samples. The recovered images from VisuShrink are overly smooth and too many coefficients are removed as a result.

SureShrink: SureShrink, follows the soft thresholding rule and is smoothness adaptive, i.e., a threshold level is assigned to each dyadic resolution level by the principle of minimizing the Stein's Unbiased Risk Estimator for threshold estimates [3].

The threshold value for SureShrink is given by:

$$th^* = \min\left(th, \sigma * \sqrt{2 * log\, n}\right) \tag{2}$$

where $th*$ denotes the value that minimizes Stein's Unbiased Risk Estimator, σ is the noise variance computed from Equation, and n is the size of the image.

4.0 STANDARD PARTICLE SWARM OPTIMIZATION

PSO is a metaheuristic, which imitates the social behaviour of natural organisms such as bird flocks and school of fishes to find a place with enough food.

The basic model consists of a swarm of N particles flying around in a D-dimensions search space, where each particle *i* is a candidate solution to the problem, and is represented by the vector *swarm(i)* in the decision space. A particle has its own position and velocity, which represents the flying direction and step of the particle. The success of some particles will influence the behavior of their neighbors. The position of each particle *swarm(i)* successively adjusts towards the global optimum depending on the two primary factors: The particle's best position $pbest_i$ denoted as $pbest_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$ and the whole swarm's best position, denoted as $gbest = (p_{g1}, p_{g2}, \ldots, p_{gD})$. The difference between the current position of the particle i and the best position of its neighbourhood is represented by the vector $(gbest_i - swarm)$.

The following operations are applied to every particle for each iteration.

4.1 Updating velocity: Velocity defines the amount of change that a particle will undergo, it is given as:

$$v_{step}(t+1) = w * v_{step}(t) + c_1 * r_1 * \left(pbest_i - swarm(t)\right) + c_2 * r_2 * \left(gbest_i - swarm(t)\right) \tag{3}$$

where w is the inertia weight, r_1 and r_2 are two random variables in the range [0, 1] and the constants c_1 and c_2 represent the cognitive and social components, respectively. They represent the attraction that a particle has either toward its own success or toward the success of its neighbors. The direction and the distance followed by the particle are defined by the velocity.

4.2 Updating position: Each particle at every iteration updates its coordinates in the decision space according to the given equation.

$$swarm = swarm + v_{step} \tag{4}$$

Then it moves to the new position.

4.3 Updating the best found particles: Each particle updates (potentially) the best local solution:

$$Iff\ swarm(i) < pbest_i, then\ pbest_i = swarm(i) \tag{5}$$

Even the best global solution of the swarm is updated:

$$Iff\ swarm(i) < gbest_i, then\ gbest_i = swarm(i) \tag{6}$$

Therefore, with every new iteration, each particle changes its position according to its own experience and that of its neighboring particles [17] [19].

5.0 HÉNON MAP BASED ADAPTIVE PARTICLE SWARM OPTIMIZATION (HAPSO)

Standard PSO fails to come out of the local optimum of the solution space because of its fast convergence rate, resulting in poor global exploration. To improve the efficiency of searching and to balance the exploration and exploitation abilities of PSO, three important improvements over standard PSO have been introduced in the proposed algorithm.

5.1 Hénon map based Particle Swarm Optimization

The control parameters r_1 and r_2 are important parameters affecting convergence of PSO. The proposed algorithm uses Hénon equations to chaotically map these control parameters thereby introducing ergodicity, irregularity and stochastic property in PSO to get efficient results. Micheal Hénon introduced this map as a simplified version of the Poincaire section of the Lorenz map [8].

The Hénon equations are given by:

$$x(t+1) = 1 + y(t) - a * \left(x(t)\right)^2 \tag{7}$$

$$y(t+1) = b * x(t) \tag{8}$$

Here, the values of a and b are set to 1.4 and 0.3, respectively, because at these values the Hénon map has a strange attractor. The values of y are normalized in the range [0, 1] and are assigned to the control parameters in the PSO equations, $r_1 = r_2 = y\,(t+1)$.

5.2 Adaptive Inertia Weight Factor (AIWF)

Inertia weight factor serves as the memory of the previous flight direction, preventing the particles from drastically changing their trajectory. The concept of linearly decreasing inertia weight factor over generations has long been introduced and modified by Shi and Eberhart [15]. However, this paper uses adaptively varying inertia weight factor to achieve an optimum tradeoff between global exploration and local exploitation. The AIWF, *w (i)*, is determined as follows:

$$w(i) = \begin{cases} w(\min) + \dfrac{(w(max) - w(min)) * (f - f(min))}{(f(avg) - f(min))}, & f \ll f(avg) \\ w(max), & f > f(avg) \end{cases} \tag{9}$$

Here, *w (min)* and *w (max)* are the minimum and maximum of *w* respectively, *f* is the current objective value of the particle, *and f (avg)* and *f (min)* are the average and minimum objective value of all the particles, respectively [6].

5.3 Adaptive Cognitive and Social Components

The cognitive component c_1 models the tendency of the particles to return to the previously found best solutions, whereas, c_2, the social component measures the performance of the particles relative to its neighbors. It is known that larger value of c_1 as compared to c_2 leads to better searching of extremes in the entire search space at the early stage of the algorithm. While a larger c_2 as compared to c_1 ensures that the particles converge quickly to the global optimum value, later.

$$c_1 = c_{1Start} - \frac{iter * (c_{1Start} - c_{1End})}{\max\_iter} \tag{10}$$

$$c_2 = c_{2Start} + \frac{iter * (c_{2End} - c_{2Start})}{\max\_iter} \tag{11}$$

Here, c_{1Start}, c_{2Start} are the initial values of the learning factors and c_{1End} and c_{2End} are the final values. *Iter* is the current iteration, and *max_iter* is the maximum number of iterations [10]. The above three approaches are then combined to give the Henon map based Adaptive Particle Swarm Optimization, described as follows:

$$\begin{aligned} v_{step}(t+1) = w(i) * v_{step}(t) + c_1 * y(t+1) \\ * \left(pbest_i - swarm(t)\right) + c_2 * y(t+1) \\ * \left(gbest_i - swarm(t)\right) \end{aligned} \tag{12}$$

where, *w (i)* is the adaptive inertia weight factor (AIWF), *y (t+1)* is the value of the normalized Hénon map sequence and c_1 and c_2 are the adaptive cognitive and social components.

Now this variant of PSO formed by combining all the above approaches, called the HAPSO is used for wavelet shrinkage image denoising.

6. WAVELET THRESHOLDING USING HAPSO

We use wavelet coefficient thresholding method for image denoising. For appropriate thresholding of coefficients several parameters have to be decided upon. They include:

- Choice of Wavelet
- Level of decomposition
- Threshold value at each level

We perform denoising of an image corrupted with Additive White Gaussian Noise (AWGN). HAPSO algorithm is employed to determine the optimum value of above mentioned parameters for denoising of noise affected images.

Parameters to be optimized	Permitted Values
Wavelet Basis (Mother Wavelet)	Daubechies (db4, db6, db8), Coiflet (coif2, coif4)
Level of Decomposition	01 to 03
Threshold value	Range is from 0.5σ to 0.9σ; σ is the estimate of noise variance. (up to three threshold values, one for each decomposition level)

Table 1: Allowed Values of Optimization Variables

Decomposition levels upto three is found to be most suitable for image denoising. PSNR is used as an evaluation criterion for measuring the effectiveness of the proposed technique.
The expression for PSNR is given as:

$$PSNR = 10 * \log_{10} \frac{G^2}{MSE} \tag{13}$$

where, G is the number of gray scale levels in the image and MSE is the mean square error between the estimated and the original image.

7. EXPERIMENTAL PROCEDURE

The main steps of HAPSO are described as follows:

Step1: Load a digital image and add additive white Gaussian noise (AWGN).

Step2: Give initial values to the parameters: *w_min, w_max, iter, max_iter,* c_{1Start}, c_{1End} c_{2Start}, c_{2End}.

Step3: Initialize the velocities and position of each particle in the 5 dimensional search space using rand function (3 threshold values; one for each level of decomposition, a wavelet basis, a level of decomposition).

Step4: Apply Hénon map sequences *(7)* and *(8)* and assign the result to the control parameters r_1 and r_2. This helps PSO to search for solution in wider space and jump out of the local minima.

Step5: Update the inertia weight factor according to equation *(9)*. The adaptively varying inertia weight factor ensures global exploration in the beginning and local exploitation towards the end.

Step6: Update the cognitive and social coefficients according to the equations *(10)* and *(11)*. This further enhances the convergence rate.

Step7: Update velocities and positions of each particle using equations *(12)* and *(4)*, respectively.

Step8: Provide the required values to the wavelet shrinkage function to obtain the denoised image. Calculate the fitness value (PSNR) from the objective function. The PSNR of the denoised image is calculated using equation *(13).*

Step9: Compare the current fitness value of the particle with its previous best value (pbest). Update the value according to the equation *(5)*.
Step10: Update the global best position in the swarm according to the equation *(10)*.
Step11: Repeat steps 5 through 10 until maximum number of iterations reached.
The corresponding flowchart is shown in figure 1.

8. SIMULATION RESULTS

The HAPSO wavelet thresholding is applied on several natural grayscale images of size 256 X 256 and the simulations are performed in MATLAB 7.6.0 environment [11] [12]. The wavelet transform employs the most optimum wavelet basis, chosen from a set of mother wavelets (Daubechie 4, Daubechie 6, Daubechie 8, Coiflet 2, Coiflet 4) using HAPSO, at three levels of decomposition, the threshold value at each level of decomposition is also optimally decided by the proposed HAPSO. In this paper the particle's search space is spread over five dimensions. To assess the performance of HAPSO, it is compared with the standard PSO, VisuShrink and SureShrink for two different test images. The adaptive parameters c_1, c_2, $w(i)$, r_1 and r_2 require their respective variables to be initialized. These variables are initialized with the values that have been found to have the most optimum effect on the proposed method.

i. $a = 1.4, b = 3$; used in equations (7) and (8)
ii. $w(min) = 0.9, \mathrm{w(max)} = 0.3$; used in equation (9)
iii. $c_{1Start} = c_{2End} = 2$; used in equations (10) and (11)
iv. $c_{2Start} = c_{1End} = 0.3$; used in equations (10) and (11)

The maximum number of iterations for all the images is kept at 50 and the swarm size of the particles also at 50. Here, PSNR is used to evaluate the performance of the proposed algorithm, represented as the cost function. Higher the value of the PSNR, the better is the effectiveness of the method used for image denoising. Equation (13) is used for evaluating the PSNR. The two cost functions graphs for 'cameraman.tif' are shown in Figure 2 and Figure 3. Figure 2 is for image denoising using standard PSO, whereas Figure 3 is for image denoising using the proposed HAPSO. On comparison, the proposed algorithm HAPSO clearly proves to be more effective than standard PSO.

The Cost function/ Fitness function becomes constant as it reaches the value 71.8 for PSO based wavelet shrinkage denoising while it reaches 74.184 for HAPSO based wavelet shrinkage denoising for 50 iterations.

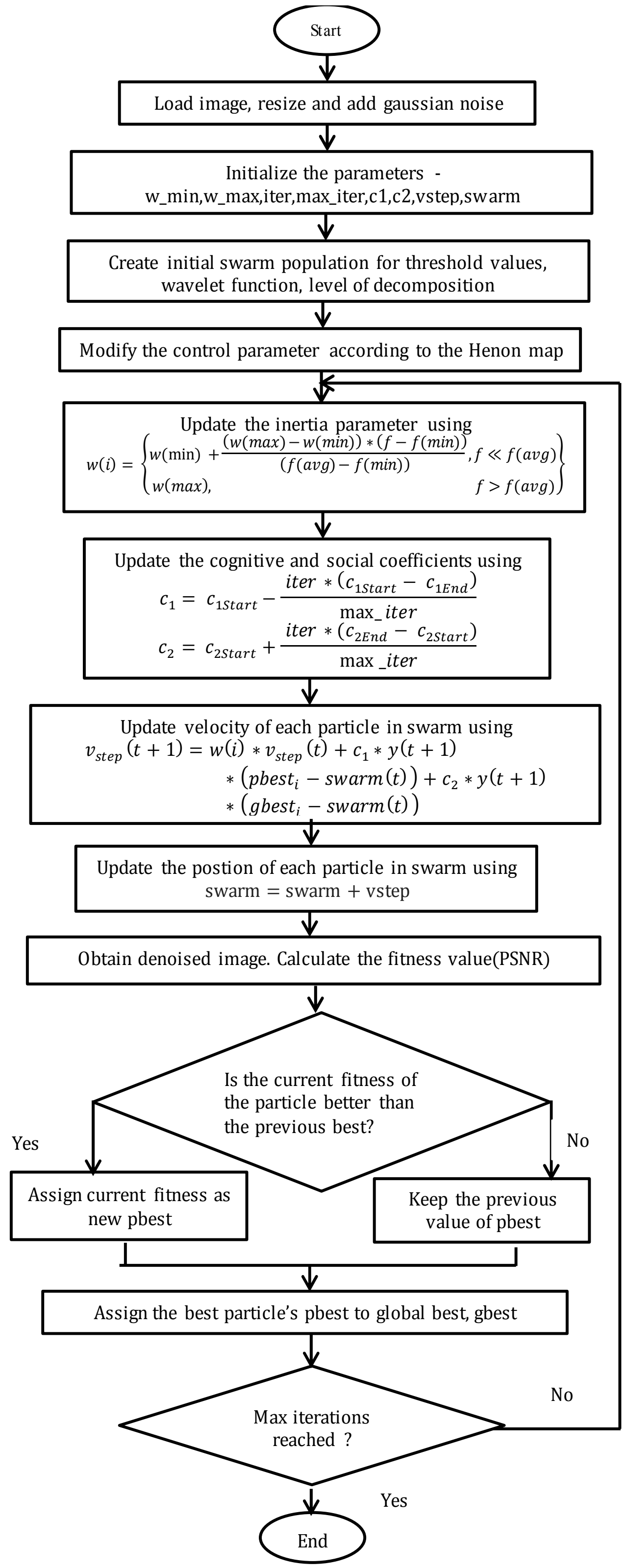


Figure 1: HAPSO Flowchart

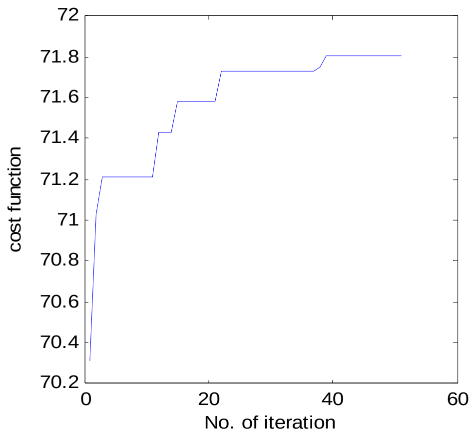


Figure 2: Cost Function for standard PSO

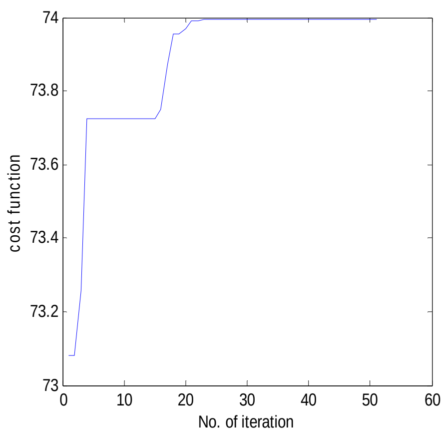


Figure 3: Cost function for HAPSO

9. CONCLUSION AND FUTURE SCOPE

In this paper, a Hénon map based adaptive Particle Swarm Optimization method for wavelet shrinkage thresholding is presented. Here we have shown that the use of HAPSO for wavelet shrinkage image denoising is more efficient when compared to the standard PSO based wavelet shrinkage denoising, and far exceeds the classical methods VisuShrink and SureShrink in terms of, not only the PSNR value but also the visual quality. Here, the HAPSO was used to optimize three wavelet parameters, the threshold value, the wavelet basis and the level of decomposition. The elements of adaptability and ergodicity introduced through Hénon equations to chaotically map the control parameters. Followed by the use of adaptive inertia weight factor (AIWF) and adaptive cognitive and social coefficients, this variant of PSO makes the process even more effective by greatly improving the global exploration of solution space, population diversity and rate of convergence. As a result, we get more optimized values of the parameters required for denoising the image using wavelet shrinkage. A tabular comparison of the simulation results and PSNR values of the denoised images as shown in Table 2, proves the effectiveness of the proposed HAPSO.

Shrinkage Methodology Used	PSNR cameraman.tif	PSNR bird.gif
VisuShrink	20.6876	22.7382
SureShrink	23.7427	26.2087
PSO based Wavelet shrinkage	71.8	72.028
HAPSO based Wavelet shrinkage	**74.184**	**77.812**

Table 2: Final Comparison of PSNR values

The work can be further extended by using modified adaptive hybrid metaheuristics combining Genetic Algorithm, PSO, Simulated annealing or their different combinations to obtain a more optimal set of wavelet thresholding parameters, and to further improve the convergence behavior as well as the quality of the denoised image.

REFERENCES

[1] D. Donoho, "De-Noising By Soft Thresholding", IEEE Transactions on Information Theory, Vol. 41, pp. 613-627, 1995.

[2] David L. Donoho and Iain M. Johnstone, "Ideal Spatial Adaptation by Wavelet Shrinkage," Biometrika, Vol. 81, No. 3, Aug., 1994, pp. 425-455.

[3] David L. Donoho and Iain M. Johnstone, "Adapting to Unknown Smoothness via Wavelet Shrinkage," Journal of American Statistical Association, 90(432):12001224, December 1995.

[4] Mallat S.G, "A Theory for Multiresolution Signal Decomposition: the Wavelet Representation," IEEE Trans. Pattn Anal. Mach. Intell., 11, 674- 693, 1989.

[5] Bhutada, G. G., R. S. Anand, and S. C. Saxena. "PSO-based learning of sub-band adaptive thresholding function for image denoising." *Signal, Image and Video Processing* 6.1 (2012): 1-7.

[6] Liu, Bo, Ling Wang, Yi-Hui Jin, Fang Tang, and De-Xian Huang. "Improved Particle Swarm Optimization Combined With Chaos." Chaos, Solitons & Fractals 25, no. 5 (2005): 1261-1271.

[7] Wang, Xuejie, Yi Liu, and Yanjun Li. "A Novel Wavelet Threshold Optimization via PSO for Image Denoising." In Computational Intelligence and Design (ISCID), 2011 Fourth International Symposium on, Vol. 1, pp. 352-355. IEEE, 2011.

[8] Coelho, Leandro dos Santos, and Viviana Cocco Mariani. "A Novel Chaotic Particle Swarm Optimization Approach using Hénon Map and Implicit Filtering Local Search for Economic Load Dispatch." Chaos, Solitons & Fractals 39, no. 2 (2009): 510-518.

[9] Du-jin Liu, Siming Li, Shuxia Sun, Zhaoyu Ding. "Application of Fast Particle Swarm Optimization Algorithm in Image Denoising". Recent Advances in Computer Science and Information Engineering

lecture Notes in Electrical engineering, Volume 126, 2012, pp. 559-566.

[10] Yongshou Dai, Hui Niu, "An Improved PSO Algorithm and its Application in Seismic Wavelet Extraction," I.J. Intelligent Systems and Applications, 2011, Vol. 5, pp. 34-40.

[11] MATLAB 7.6.0, Image Processing Toolbox.

[12] MATLAB 7.6.0, Wavelet Toolbox.

[13] M. Antonini, M. Barlaud, P. Mathieu, and I.Daubechies, "Image Coding using Wavelet Transform," IEEE Trans. Image Processing, vol. 1, no. 2, pp. 205– 220, 1992.

[14] Rafael C. Gonzalez and Richard E. Woods – Digital Image Processing; Second Edition; Pierson Prentice Hall, 2007.

[15] Yuhui Shi and Russell Eberhart, Indiana University Purdue University Indianapolis, "*A Modified Particle Swarm Optimizer*", 1998.

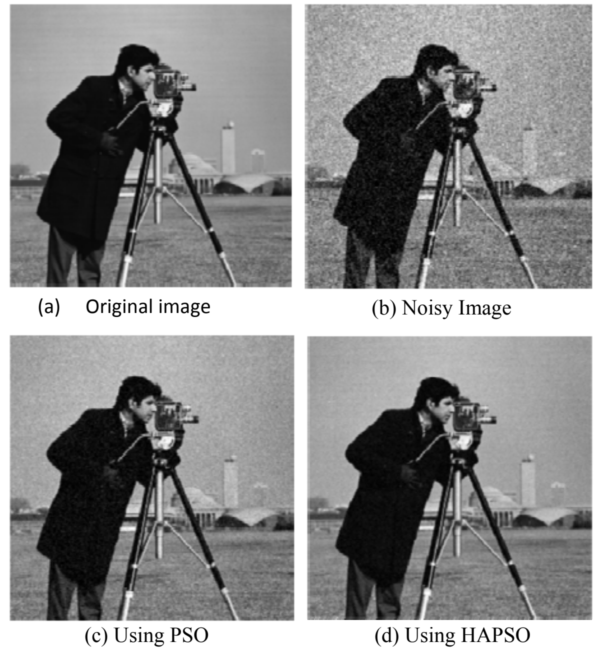
[16] Magnus Erik Hvass Pedersen, Hvass Laboratories, Technical Report no. HL1001, "Good Parameters for Particle Swarm Optimization," 2010.

[17] El-Ghazali Talbi , "Metaheuristics from Design to Implementation", University of Lille – CNRS – INRIA

[18] M. Antonini,M. Barlaud,P. Mathieu, and I.Daubechies, "Image Coding Using Wavelet Transform," IEEE Trans. Image Processing, vol. 1, no. 2, pp. 205– 220, 1992.

[19] Kennedy, J. and Eberhart, R., "Particle Swarm Optimization," Proceedings of the IEEE International Conference on Neural Networks, Perth, Australia 1995, pp. 1942-1945.

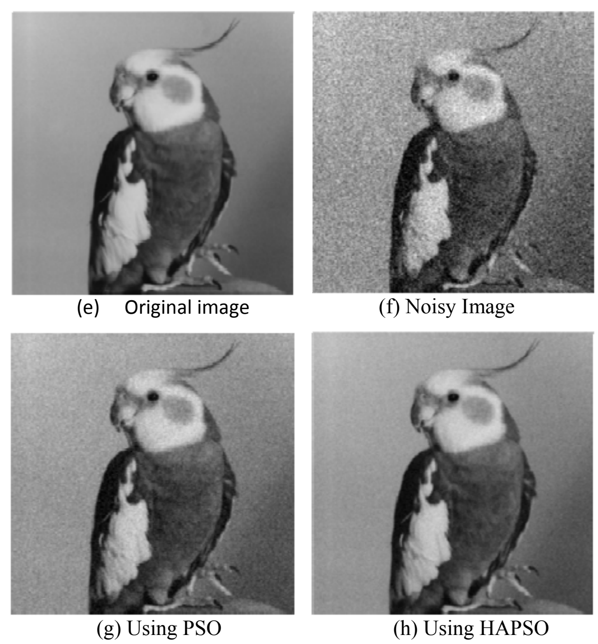
[20] Chan-Cheng Liu, Tsung-Ying Sun, Shang-Jeng Tsai, Yu-Hsiang Yu, Sheng-Ta Hsieh, Heuristic wavelet shrinkage for denoising, Applied Soft Computing, Volume 11, Issue 1, January 2011, Pages 256-264



(a) Original image (b) Noisy Image

(c) Using PSO (d) Using HAPSO

Figure 4: Simulations performed on 'cameraman.tif'



(e) Original image (f) Noisy Image

(g) Using PSO (h) Using HAPSO

Figure 5: Simulations performed on 'bird.gif'